respectively be 129 times the total number of combinations of n things taken 1, 2, 3 .. ..n respectively, find n.

(*c*) Prove that the number of permutations of $2n$ things of which one-half are alike and the other half alike though different from the first is equal to the greatest number of combinations that can be made of the $2n$ things of which none are alike.

6. (*a*) Book work.

(*b*) The series formed by writing down the terms is obviously the expansion of $2^{2n} - 1$ in one case and of $2^n - 1$ in the other. We have

$$\therefore\ 2^{2n} - 1 = 129\,(2^n - 1),$$
$$\text{or } 2^{2n} - 129.\ 2^n + 128 = 0,$$
$$\text{or } (2^n - 1)\,(2^n - 128) = 0;$$
$$\therefore\ n = 0 \text{ or } 7.$$

(*c*) Book work.

7. (*a*) Deduce the formula for the sum of n terms of a Geometrical Progression.

(*b*) If a G. P. whose ratio is r consists of $4n$ terms, show that the sum of the first and last n terms is to the sum of the second and third n terms as $r^{2n} - r^n + 1 : r^n$.

(*c*) If x, y, z be in H. P., a, x, b, in A. P. and a, z, b, in G, P., prove that

$$y = 2\,(a+b)\left\{\left(\frac{a}{b}\right)^{\frac{1}{4}} + \left(\frac{b}{a}\right)^{\frac{1}{4}}\right\}^{-2}.$$

7. (*a*) Book work.

(*b*) Sum of the 1st and 4th n terms is

$$\frac{a(r^n - 1)}{r - 1} + \frac{ar^{3n}(r^n - 1)}{r - 1}.$$

Sum of the 2nd and 3rd n terms is

$$\frac{ar^n\,(r^n - 1)}{r - 1} + \frac{ar^{2n}\,(r^n - 1)}{(r - 1)}$$

$\therefore$ the ratio of these sums =

$$\frac{r^{3n} + 1}{r^n\,(r^n + 1)} = \frac{r^{2n} - r^n + 1}{r^n}.$$

(*c*) We have $y = \dfrac{2\,x\,z}{x+z} = \dfrac{2\,(a+b)\sqrt{ab}}{a+b+2\sqrt{ab}}$

$$= 2\,\frac{(a+b)\,\sqrt{ab}}{(\sqrt{a}+\sqrt{b})^2}$$

$$= 2\,(a+b)\left(\frac{a^{\frac{1}{4}}}{b^{\frac{1}{4}}} + \frac{b^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)^{-2}$$

8. (*a*) Define the terms ratio and proportion, and explain what is meant by the statement that one quantity varies inversely as another.

(*b*) If $x \propto y$, and $xy \propto z^2$, prove that $x^3 + y^3 + z^3 \propto xyz$.

(*c*) Shew that any sum of money, twice the discount on it for a given time and the interest on the sum for the same time are in harmonical proportion.

8. (*a*) Book work.

(*b*) $x = py$ and $ny = qz^2$, from which $y = \left(\dfrac{q}{p}\right)^{\frac{1}{2}} z$ and $x = (pq)^{\frac{1}{2}}\,z$,

and $x^3 + y^3 + z^3 = \left\{(pq)^{\frac{3}{2}} + \left(\dfrac{q}{p}\right)^{\frac{3}{2}} + 1\right\} z^2$,

and $x\,y\,z = qz^3$; $\therefore\ x^3 + y^3 + z^3 \propto x\,y\,z$

(*c*) Let n be the sum of money, r the rate, and t the time. The interest $= x, r, t$, and the discount $= \dfrac{xrt}{1+rt}$, and

$\therefore\ \dfrac{1}{x}\ .\ \dfrac{1+rt}{2\,xrt}$ and $\dfrac{1}{xrt}$ are in A, P, $\therefore$ etc.

9. (*a*) The sum of p terms of an Arithmetical Progression is q and the sum of q terms is p, show that the sum of $(p - q)$ terms is $\left(\dfrac{2q}{p} + 1\right)(p - q)$.

(*b*) Shew that the sum to n terms of the series, $2 + 2.3 + 2.7 + 2.15 + \ldots\ldots + (2^{n+1} - 2) = 2^{n+2} - 2\,(n+2)$.

(*c*) If A be the sum of the series formed by taking the 1st and every p^{th} term after the first of an infinite Geometrical Progression whose first term is one and whose ratio is less than one, and if B be the sum of the series formed by taking the 1st and every q^{th} term after the first, prove

$$A^q\,(B - 1)^p = B^p\,(A - 1)^q.$$

9. (*a*) $S_p = \dfrac{p}{2}\left\{2\,a + (p - a)\,b\right\} = q$,

and $S_q = \dfrac{q}{2}\left\{2\,a + (q - 1)\,b\right\} = p$,

$\therefore\ a = \dfrac{p^2 + pq + q^2 - p - q}{pq}$ and b

$$= -\,\frac{2\,(p+q)}{pq},$$

$\therefore\ S_{p-q} = \dfrac{p-q}{2}\left\{2\,a + (p - q - 1)\,b\right\}$

$= (p - q)\left(\dfrac{2q}{p} + 1\right)$ by reduction.