3. Ceva's Theorem:—If from the vertices of a triangle concurrent straight lines be drawn to cut the opposite sides, the product of one set of alternate segments taken in circular order is equal to the product of the other set.

(NOTE.—D, E *and* F *must be on the three sides, or on one side and on the other two produced.*)

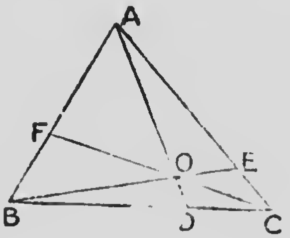

FIG. 5.

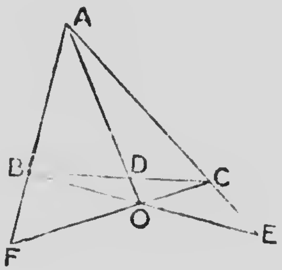

FIG. 6.

AO, BO, CO drawn from the vertices of $\triangle$ ABC cut BC, CA, AB, at D, E, F respectively, then

$$AF \cdot BD \cdot CE = FB \cdot DC \cdot EA.$$

FOC is a transversal of $\triangle$ ABD,

$\therefore$ $AF \cdot BC \cdot DO = FB \cdot CD \cdot OA$. (§ 1.)

BOE is a transversal of $\triangle$ ADC,

$\therefore$ $AO \cdot DB \cdot CE = OD \cdot BC \cdot EA$.

By multiplication, and division by DO, OA and BC,

$$AF \cdot BD \cdot CE = FB \cdot DC \cdot EA.$$

(For another proof of this theorem see O. H. S. Geometry, § 122, Exercises 12 and 14.)

(HISTORICAL NOTE.—Giovanni Ceva, an Italian engineer, died in 1734 A.D.)