Weather during month equable and delightfully pleasant; the most tender plants have not been checked or injured. Cucumbers, melons and tomatoes still growing; no equinoctial weather; barometric changes very small all through the month.

BELLEVILLE.—Lightning and thunder, with rain, 1st, 2nd, 9th, 15th. Rain, 1st, 2nd, 3rd, 5th, 9th, 15th, 16th, 23rd, 24th, 30th. Streamers and corona observed in aurora of 26th. Barometer steady during month.

GODERICH.—On 5th, large lunar halo at 9 P.M. On 8th, lightning. 15th, thunder, with rain. 1st, 7th, 9th, lightning and thunder, with rain. Wind storms, 1st, 9th. Rain, 1st, 2nd, 3rd, 4th, 7th, 9th, 15th, 23rd, 24th, 25th, 29th, 30th. Month remarkable for steadiness and height of barometer; absence of winds; very fine, clear, pleasant weather.

SIMCOE.—Fogs, 26th, 28th. Rain, 3rd, 5th, 7th, 8th, 16th, 24th, 29th, 30th. Beautiful weather, but much sickness; an unusual degree of malarious disease.

STRATFORD.—On 7th, thunder with rain. 15th, lightning, with thunder. Frost, 12th, 13th, 20th. Fogs, 10th, 12th, 24th, 26th, 28th, 29th. Rain, 3rd, 7th, 15th, 16th, 23rd, 24th, 25th, 29th, 30th. The frosts this month were very slight, doing no damage to vegetation.

HAMILTON.—On 1st lightning. 9th, lightning and thunder, with rain. 15th, rainbow at 4 P.M. 20th, auroral arch, streamers, and crimson vapour. 25th, a number of arches, dipping towards W, streamers and crimson vapour. Wind storm on 16th. Fogs, 13th, 16th. Rain, 3rd, 5th, 9th, 15th, 16th, 23rd, 24th, 25th, 29th, 30th. Splendid weather during whole month.

WINDSOR.—On 2nd and 15th, lightning, Lunar halo on 2nd, 3rd, 4th, 5th. Meteors on 4th, 7th, 12th (2), 17th (2), 18th (4), 20th, 21st, 27th. Frost on 5th. Fog, 21st. Rain, 3rd, 9th, 24th, 29th, 30th.

III. Intercommunications with the "Journal."

1. PROOF OF THE GEOMETRICAL THEOREMS OF W.

J. C. GLASHAN.

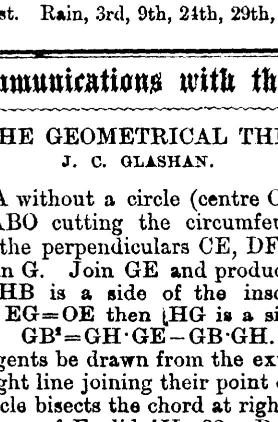
"From a point A without a circle (centre O) draw the tangent AC, and the line ABO cutting the circumference in B. Bisect AC in D. Let fall the perpendiculars CE, DF on AO. Draw FG touching the circle in G. Join GE and produce to meet the circle in H. Then (1) HB is a side of the inscribed square. (2) $HB^2 = HE \cdot HG$. If EG=OE then HG is a side of the inscribed equilateral triangle. $GB^2 = GH \cdot GE - GB \cdot GH$.

Lemma I. If tangents be drawn from the extremities of a chord of a circle, the straight line joining their point of intersection with the centre of the circle bisects the chord at right angles.

Lemma II. Converse of Euclid III. 22. Prove by *reductio ad absurdum.*

Definition.—Join any point with the centre of a circle, and take a point on the joining line such that the rectangle under the distances of it and of the given point from the centre shall be equal to the square on the radius. The perpendicular through the assumed point to the joining line is called the *polar* of the given point which is called the *pole.*

Lemma III. A chord is drawn through a fixed point and tangents at its extermities, the locus of their intersection is the polar of the fixed point.

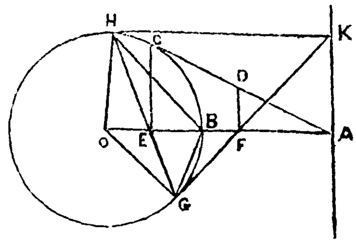


Let C be the centre of the circle and P the fixed point, and let the tangents AD, BD at the extremities of the chord AB passing through P meet in D.

Through D draw DO perpendicular to CP (produced if necessary). Join CD, bisecting AB at right angles, in E (Lem. I.) and DAC is a right angle $\therefore DC \cdot CE = AC^2$ (Euc. VI. 8 cor. and 17). And the angles POD, DEP are right $\therefore$ the points D, E, P, O lie on the same circumference (Lem. II.) $\therefore DC \cdot CE = OC \cdot CP$ (Euc. III. 36 cor.) $\therefore OC \cdot CP = AC^2$ $\therefore$ DO is the polar of P, that is, the point D always lies on the fixed straight line, the polar of P.

Corollary.—If DO equals AC, then is ACO a right angle. The points A, C, O, D lie in the circumference of a circle (Lem. II.) $\therefore$ if DO equals AC, the angle DCO equals ADC (Euc. III. 27) $\therefore$ the angle ACO is a right angle (Euc. I. 32).

Lemma IV. The diagonals of a square bisect each other at right angles, and conversely.



Theorems.—Construct as directed. (1.) Through A draw AK perpendicular to AO, AK is the polar of E for the circle HCB (Lem. III.), for A is the intersection of tangents from the extremities of the chord of which CE is the half and AK is perpendicular to OD produced. Produce GF to intersect the tangent from H in K a point in AK (Lem: III.) Join HO, OG. AC is bisected in D $\therefore AF = FE$ (Euc. VI. 2) $\therefore AO \cdot OE = OE^2 + 2OE$. EF (Euc. II. 3 and 1). $AO \cdot OE = OG^2$ (def.) $\therefore OE^2 + 2OE \cdot EF = OG^2$ $\therefore OF^2 = OG^2 + EF^2$ (Euc. II.) $= OG^2 + EF^2$ (Euc. III. 18, I. 47) $\therefore GF = EF = FA$ $\therefore AK = OG$ (Euc. I. 26) $= HO$ $\therefore$ the angle HOA is a right angle (Cor., Lem. III.) $\therefore$ HB is the side of the inscribed square (Lem. IV.).

(2.) Join GB. The angles HGB and HBE are equal (Euc. III. 20) $\therefore HB^2 = HE \cdot HG$ (Euc. VI. 4 and 17).

(3.) OE=EG $\therefore$ the angle OGE = angle GOE (Euc. I. 5). The angle FGE = angle FEG (1st and Euc. I. 5) = twice angle OGE $\therefore$ thrice angle OGE = a right angle (Euc. III. 18) $\therefore$ angle FGE $= \frac{2}{3}$ of a right angle $\therefore$ HG is a side of the inscribed equilateral triangle (Euc. III. 32).

(4.) Produce HB to L so that BL = BG. Join GL and from G let fall GN perpendicular to BL. The angle HBG $= \frac{4}{3}$ right angle (3rd and Euc. III. 22) $\therefore$ triangle BGL is equiangular (Euc. I. 13, 32, 5) $\therefore 2BN = BL = BG$ (Euc. I. 26). $HG^2 = HG \cdot GE + HB^2$ (Euc. II. 2 and 2nd). $HG^2 = HB^2 + BG^2 + 2BN \cdot BH$ (Euc. II. 12) $= HB^2 + BG^2 + GB \cdot BH$ $\therefore GB^2 = HG \cdot GE - GB \cdot BH$. *Q. E. D.*

Lemma III. is an important proposition in the theory of polars, a theory which, by a set of some half dozen propositions, all as simple as the Lemma, doubles, as it were, the propositions of geometry; that is, any theorem of position being admitted to be sure, the theory of polars immediately gives another as true. If W., or some other reader of the *Journal,* would give a few elementary articles on The Modern Geometry (synthetic) I believe they would be most interesting to many teachers who have "kept to Euclid." Meanwhile, I propose several thorems well known in the polar theory, and not very difficult to be proved by the Ancient Geometry. Those who follow the latter will, where necessary, translate the enunciation into Euclid's language. To facilitate this I give two enunciations of the first. It was proposed by Sexell Nova Acta Petrop. 1780.

(1.) ABCD is a quadrilateral inscribed in a circle whose centre is O, and the opposite sides are produced to meet, viz.: AB and CD in E, and BC and AD in F; join CO and FO, meeting the circle in R and S; take OP : O R : : OR : OE, and OQ : OS : : OS : OF, then if EQ, FP be drawn they will be perpendicular to FO, EO :—

Or, if a quadrilateral be inscribed in a circle the pole of each extremity of the *third* diagonal passes through the other extremity.

(2.) If a quadrilateral be inscribed in a circle, and another circumscribed, touching at the angular points, their diagonals intersect in the same point: their *third* diagonals are in the same straight line; the intersection of each pair of their *three* diagonals is the pole of the remaining one; and the intersection of their diagonals and the extremities of the *third* diagonal of the inscribed quadrilateral are each the pole of the line joining the other two of these points.

(3.) If a quadrilateral be inscribed in a circle and the figure completed, the square on the third diagonal is equal to the sum of the squares on the two tangents from its extremities, the tangents from the middle point of the third diagonal are each equal to half the diagonal, and the circle on the third diagonal as diameter cuts the given circle orthogonally.

(4.) The circles circumscribing the four triangles of a complete quadrilateral all pass through the same point, and this point and the four centres all lie on the same circumference.

Def.—If the two pairs of opposite sides of a quadrilateral be produced to intersect, the straight line joining the points of intersection is called the *third diagonal* of the figure which is called a *complete quadrilateral.*

Def.—If two circles intersect and the tangents at the points of intersection be at right angles, the circles are said to cut *orthogonally,*