$$\therefore \frac{5}{x}+\frac{4}{y}+\frac{6}{z}=3\frac{52}{60} \quad (a)$$

$$\frac{6}{x}+\frac{4}{y}+\frac{5}{z}=4 \quad (b)$$

$$\frac{5}{x}+\frac{5}{y}+\frac{5}{z}=3\frac{55}{60} \quad (c)$$

Multiply (a) by 5, (b) by -10, (c) by 4, and add these results, and we have $x=3$. Then $y=4$, $z=5$.

8. (i)

$$(x^2-2x+5)+6\sqrt{x^2-2x+5}=16$$

$$\therefore \sqrt{x^2-2x+5}=3\pm 5$$

$$\therefore x^2-2x+5=64 \text{ or } 4$$

$$\therefore x=1,\ 1,\ 1\pm\sqrt{60}$$

(ii) $x^4+y^4=257 \quad (a)$

$x+y=5 \quad (b)$

substitute $m+n$ for x

and $m-n$ for y

Then (a) becomes

$$2m^4+2n^4+12m^2n^2=257 \quad (c)$$

and (b) $2m=5$

substituting for m in (c) we get

$$16n^4+600n^2=1431$$

$$\text{whence } n=\pm\frac{3}{2},\ \pm\frac{1}{2}\sqrt{-159}$$

$$\therefore x=m+n=4, \&c.$$

$$y=m-n=1, \&c.$$

(iii)

$$a^x b^y c^z = l$$

$$a^y b^z c^x = m$$

$$a^z b^x c^y = n$$

whence

$\log. a \,.\, x+\log. b \,.\, y+\log. c \,.\, z=\log. l$

$\log. c \,.\, x+\log. a \,.\, y+\log. b \,.\, z=\log. m$

$\log. b \,.\, x+\log. c \,.\, y+\log. a \,.\, z=\log. n$

$$\text{whence } x=\frac{l(bc-a^2)+m(ab-c^2)+n(ca-b^2)}{3abc-a^3-b^3-c^3}$$

writing a for log. a, &c.

11. Take O, the centre, and join OP, OQ. Let the tangents at P, Q meet at K. Then OPRQ is a quadrilateral, having the angles at P, Q right angles; therefore the angles POQ, PRQ are together equal to two rt. angles; hence the angle POQ is equal QRK (formed by producing PR to K).

ALGEBRA.

1. $$a^{-2}-2+a^2=\frac{1-2a^2+a^4}{a^2} \&c.$$

$\therefore$ the expression becomes

$$\frac{\dfrac{1-2a^2+a^4}{a^2}\times\dfrac{1-2a+a^2}{a}\times a^2}{1-a^4-2a+2a^3}$$

$$=\frac{(1-a^2)^2(1-a)^2}{a(1-a^2)(1+a^2-2a)}$$

$$=\frac{1-a^2}{a}=a^{-1}-a$$

2. $$\frac{1+x}{1-x}(1+x+2x^2+x^3)$$

3. The expression when multiplied out becomes

$$2s^3-(a+b+c)s^2+abc$$

$$=2s^3-(2s)s^2+abc$$

$$=abc$$

4. $x+p+q$ is a measure of x^2+px+p^2 if $(p+q)^2-p(p+q)+p^2=0$

that is if $p^2+pq+q^2=0 \quad (a)$

Similarly $x+p+q$ can be shewn to be a measure of x^2+qx+q^2

But (a) is also the condition that $x-p$ may be a factor of x^2+qx+q^2, and that $x-q$ may be a factor of x^2+px+p^2

$\therefore x^2+px+p^2=(x-q)(x+p+q)$

and $x^2+qx+q^2=(x-p)(x+p+q)$

and the L. C. M. of these is

$$=(x-p)(x-q)(x+p+q)$$

$$=(x-p)(x^2+px+p^2)$$

$$=x^3-p^3$$

5. Book work.

6. (1) $$\frac{a}{x-a}+\frac{b}{x-b}+\frac{c}{x-c}$$

$$=\frac{abc}{(x-a)(x-b)(x-c)}$$

becomes $(a+b+c)x^2$

$-2(ab+bc+ca)x+2abc=0$

$$\therefore x=\frac{ab+bc+ca\pm\sqrt{a^2b^2+b^2c^2+c^2a^2}}{a+b+c}$$