

Debt Repayment by Equal Annual Instalments

Instalment Debentures Make Sinking Fund Unnecessary—Advantages to Municipality More • Than Compensate for Any Difference in Price Obtained—Mathematical Difficulties Sometimes Prevent Use of This Form—A Simple Method of Calculating Amount of Instalments

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WITH the exception of cases where very long term bonds are issued on securities that have more or less unchangeable value and where it is contemplated that new bonds will be issued to take care of the old ones when they fall due, two general methods of financing are usually followed. First, the individual usually gives a mortgage on some security, under an agreement that he will pay a certain amount of the liability each year with interest at the agreed rate and make final settlement at the end of the term of the mortgage; the second method followed by municipalities, cities and small corporations is to sell bonds bearing an agreed rate of interest payable annually, the amount of the bond becoming due at the end of a limited number of years, usually varying between 15 and 30. The municipality, city or corporation prepare themselves to meet the obligation by creating a sinking fund, that is, levying an amount of tax or setting aside an amount out of the business as the case may be, which, put out at interest (usually at a rate considerably lower than the rate of interest they are compelled to pay on the bond), so that at the end of the given period the fund thus created will be sufficient to meet the obligation.

The writer believes that a third method-namely, of liquidating the debt by equal annual payments, would in general be more satisfactory and better adapted to rural municipalities and small business concerns, who, in a great many cases, do not have the facilities or the knowledge to properly handle a sinking fund. This latter method would have the advantage that the taxpayer or small business man would not in general be penalized on account of the different rate of interest they would have to pay and what they would receive on a sinking fund, and the writer believes further that such bonds would in general command about as high a price as the bond requiring a single payment at the end of the term of the life of bond, for the reason that the liability would be constantly decreasing, therefore the security of the bondholder would always be getting safer and it would be only carrying out a principle that is usually insisted on by trust and mortgage companies when making loans to individuals.

Calculating Amount of Payments

In discussing this matter with the officials of a rural municipality, the writer was told by the officials that they would like to issue such debentures, but they did not know how to figure out the amount of the annual payments. This may be the reason why this form of debenture is not more popular. The problem, however, is not a very difficult one, for the reason that the percentage of the debt to be paid each year is equal to the amount of \$1 for the given number of years, at the agreed rate of interest compounded, divided by the amount of \$1 per year for the given number of years, at the agreed rate of interest compounded, that is, if we let "R" equal the rate of interest and "N" the number of annual payments, then "P" the percentage of the debt to be paid each year, will equal

$$P = \underbrace{\frac{(1+R)^{n}}{(1+R)^{n}-1}}_{R} \qquad (1)$$

this can be reduced to

The terms of the numerator and the denominator of equasion (1) can be taken from compound interest tables and dividing the numerator by the denominator, "P" can be found to a value extending to the fifth or sixth decimal place. The converse of this problem is not such an easy one. That is, if you had given a number of equal annual payments to liquidate a debt of a given sum, you could find the value of "P" by dividing an annual payment by the total sum but having given "P," it is not so easy to find out the value of "R," that is, the rate of interest that you are actually paying.

Illustration of Method

To facilitate and explain this matter in general the writer has prepared a chart, which gives in the table the percentage of the total debt to be paid each year in 10, 15, 20, 25 and 30 annual payments, at rates of interest varying in even percentages from 2 per cent. to 10 per cent. By plotting this table as shown in the chart, we have a means of determining "R." Having given "P," and the number of equal annual payments, it is very easy to determine within a very small fraction, the rate of interest that is being paid.

For instance, if you are paying \$1,600 per year to liquidate a debt of \$20,000, the percentage paid each year would be 8 per cent. of the total debt. If the agreement called for 15 equal annual payments, you would be paying interest, approximately, at the rate of 2.4 per cent. If it was 20 equal annual payments, you would be paying interest at the rate of 4.94 per cent., 25 equal annual payments, interest would be 6.23 per cent., and if it was 30 equal annual payments the interest would be 6.94 per cent.

Take another example. Suppose a municipality sells a 20-year 5 per cent. bond at par and arrange to take care of the debt by a sinking fund estimated on a basis of 3 per cent. compound interest. For every dollar sold the taxpayer will have to pay each year for 20-years, 5 cents as interest and 3.72 cents to sinking fund, or a total of 8.72 cents, which is the equivalent of paying about 6 per cent. for money, while if equal payment method was used, only 8.03 cents per dollar would have to be collected. On a basis of the above assumptions, the municipality could afford

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