# UNIVERSITY WORK.

## MATHEMATICS.

ARCHIBALD MACMURCHY, M.A., TORONTO, EDITOR.

## EDUCATION DEPARTMENT, ONTARIO.

JULY EXAMINATIONS, 1883.

*Second Class Teachers.*

ALGEBRA.

Solutions by J. L. Cox, B.A., Mathematical Master, Coll. Inst., Collingwood.

1. (1) If $x^2 - mx + 1 = 0$, express

$\frac{1}{x^2}(x^4 - 3x^3 + 2x^2 - 3x + 1)$ as a function of $m$

(2) If $x + y = m$, and $xy = n$, express $x^2 + y^2$, and $\frac{1}{x^2} + \frac{1}{y^2}$ in terms of $m$ and $n$.

1. (1) $x^2 - mx + 1 = 0$, $\therefore\ x^2 + 1 = mx$.

$$\frac{1}{x^2}\left\{x^4 + 2x^2 + 1 - 3x(x^2 + 1)\right\} =$$

$$\frac{1}{x^2}\left\{(x^2 + 1)^2 - 3x(x^2 + 1)\right\} =$$

$$\frac{1}{x^2}\left\{m^2x^2 - 3x \cdot mx\right\} = m^2 - 3m.$$

(2) $x^2 + y^2 = (x+y)^2 - 3xy(x+y) = m^2 - 3mn$,

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2 + y^2}{x^2y^2} = \frac{(x+y)^2 - 2xy}{x^2y^2} = \frac{m^2 - 2n}{n^2}.$$

2. If $cx + sy = \sqrt{a^2c^2 + b^2s^2}$

$- sx + cy = \sqrt{a^2s^2 + b^2c^2}$

and $c^2 + s^2 = 1$,

then $x^2 + y^2 = a^2 + b^2$.

2. $c^2x^2 + s^2y^2 + 2csxy = a^2c^2 + b^2s^2$

$s^2x^2 + c^2y^2 - 2csxy = a^2s^2 + b^2c^2$

$\therefore\ x^2(c^2 + s^2) + y^2(c^2 + s^2) = a^2(c^2 + s^2) + b^2(c^2 + s^2)$;

$\therefore\ x^2 + y^2 = a^2 + b^2$.

3. If $a$, $b$ be integers, and $\frac{a}{b} = \frac{3}{4}$, then $a$ is a multiple of 3, and $b$ is the same multiple of 4.

3. Bookwork.

4. (1) Simplify

$\sqrt{x^6 - y^6}\sqrt{x^6 + y^6}\sqrt{x^6 - y^6}\sqrt{x^6 + x^3y^3 + y^6}$.

(2) Extract the square root of

$$\frac{x^2}{9y^2} + \frac{2x}{3y} + \frac{11}{9} + \frac{2y}{3x} + \frac{y^2}{9x^2}.$$

4. (1) $\sqrt{x^6 - y^6}\sqrt{x^6 + y^6}\sqrt{x^6 - y^6}$

$$\sqrt{x^6 + x^3y^3 + y^6}$$

$$= \sqrt{(x-y)(x+y)(x+y)(x^2 - xy + y^2)(x-y)}$$

$$\sqrt{(x^2 + xy + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)}$$

$$= (x-y)(x+y)(x^2 - xy + y^2)(x^2 + xy + y^2)$$

$$= x^6 - y^6.$$

(2) Sq. root $= \frac{x}{3y} + 1 + \frac{y}{3x}$.

5. Solve the equations

(1) $x + \frac{24}{3x+1} = 2x - 3\frac{1}{2}$.

(2) $6x + \sqrt{x} = 2$.

5. (1) $\frac{24}{3x+1} = x - \frac{7}{2}$,

$\therefore\ 24 = 3x^2 + x - \frac{21x}{2} - \frac{7}{2}$,

$\therefore\ 48 = 6x^2 - 19x - 7$, $6x^2 - 19x - 55 = 0$,

$(6x + x)(x - 11) = 0$, $x = -\frac{11}{6}$ or 5.

(2) $6x + \sqrt{x} - 2 = 0$,

$\sqrt{x} = \frac{-1 + \sqrt{1 + 48}}{12} = \frac{-1 \pm 7}{12} = \frac{1}{2}$ or $-\frac{2}{3}$,

$\therefore\ x = \frac{1}{4}$ or $\frac{4}{9}$.

6. Solve the equations

(1) $x - y = 3$, $xy = 18$.

(2) $x^2 - xy = 3$, $x^2 - y^2 = 5$.

(3) $x - y = a$, $y - z = b$, $z + x = c$.

6. (1) $x - y = 3$ $xy = 18$, $x - \frac{1}{x} = 3$.

$x^2 - 3x - 18 = 0$,

$(x - 6)(x + 3) = 0$,

$x = 6$ or $-3$,

$y = 3$ or $-6$.

(2) $x^2 - xy = 3$, (1) $\frac{(1)}{(2)}$ $\frac{x}{x+y} = \frac{3}{5}$,

$x^2 - y^2 = 5$, (2)

$5x = 3x + 3y$,

$2x = 3y$,

$\frac{2}{3}x = y$,

$x^2 - \frac{4}{9}x^2 = 5$, and $x = \pm 3$, $y = \pm 2$.