$$\therefore S_n = \frac{1}{(x+y+z)(x+2y+3z)} - \frac{1}{(x+y+z)\left\{(n+1)x+(n+2)y+(n+3)z\right\}}$$

$$= \frac{n}{(x+2y+3z)\left\{(n+1)x+(n+2)y+(n+3)z\right\}}.$$

186. Sum

$$1 - \frac{3}{4} + \frac{3.5}{4.8} - \frac{3.5.7}{4.8.12} + \text{\&c. to infinity.}$$

$$1 - \frac{3}{4} + \frac{3.5}{4.8} - \frac{3.5.7}{4.8.12} + \ldots$$

$$= \left(1+\frac{1}{2}\right)^{-\frac{3}{2}} = \left(\frac{3}{2}\right)^{-\frac{3}{2}} = \left\{\frac{1}{\frac{3}{2}}\right\}^{\frac{3}{2}} = \left(\frac{2}{3}\right)^{\frac{3}{2}}.$$

Solutions by proposer, W. S. ELLIS, B.A., Math. Master, Coll. Inst., Cobourg.

176. Apply the principles of algebraic expansion and factoring to the solution of the following arithmetical problems:—

Simplify

(a) $\dfrac{\frac{1}{4} - \frac{1}{3} + \frac{1}{9}}{\frac{1}{2} - \frac{1}{3}}$; (b) $\dfrac{(\frac{3}{8}+\frac{1}{3})^2 - \frac{3}{8} - \frac{1}{3} + \frac{1}{4}}{\frac{3}{8}+\frac{1}{3}-\frac{1}{2}}$;

(c) $\dfrac{2499}{49}$; (d) $\dfrac{6364}{15\times 5+11}$;

(e) $\dfrac{\frac{8}{27}+1+\frac{9}{8}+\frac{27}{64}}{(\frac{2}{3}+\frac{3}{4})^2}$; (f) $\dfrac{(\frac{1}{2})^4 + 2(\frac{1}{2}\times\frac{1}{3})^2 + (\frac{1}{3})^4}{\frac{1}{4}+\frac{1}{9}}$;

(g) $\dfrac{.5\times.25-.25\times.0625}{.5-(.5)^2}$; (h) $\dfrac{2\times4\times8-.5\times.25}{5-1.5}$

(a) may be thrown into the form

$\dfrac{(\frac{1}{2}-\frac{1}{3})^2}{\frac{1}{2}-\frac{1}{3}}$, the quotient is $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$.

(b) $$\frac{(\frac{3}{8}+\frac{1}{3})^2-\frac{3}{8}-\frac{1}{3}+\frac{1}{4}}{\frac{3}{8}+\frac{1}{3}-\frac{1}{2}} = \frac{(\frac{3}{8}+\frac{1}{3}-\frac{1}{2})^2}{\frac{3}{8}+\frac{1}{3}-\frac{1}{2}}$$
$$= \frac{3}{8}+\frac{1}{3}-\frac{1}{2} = \frac{1}{3}-\frac{1}{8} = \frac{5}{24}.$$

(c) $$\frac{2499}{49} = \frac{50^2-1}{50-1} = \frac{(50+1)(50-1)}{50-1}$$
$$= 50+1 = 51.$$

(d) $$\frac{6364}{15\times5+11} = \frac{6400-36}{86}$$
$$= \frac{(80+6)(80-6)}{80+6} = 74.$$

(e) $$\frac{\frac{8}{27}+1+\frac{9}{8}+\frac{27}{64}}{(\frac{2}{3}+\frac{3}{4})^2} = \frac{(\frac{2}{3}+\frac{3}{4})^3}{(\frac{2}{3}+\frac{3}{4})^2} = \frac{2}{3}+\frac{3}{4} = 1\tfrac{5}{12}.$$

(f) $$\frac{(\frac{1}{2})^4+2(\frac{1}{2}\times\frac{1}{3})^2+(\frac{1}{3})^4}{\frac{1}{4}+\frac{1}{9}} = \frac{\{(\frac{1}{2})^2+(\frac{1}{3})^2\}^2}{(\frac{1}{2})^2+(\frac{1}{3})^2}$$
$$= \frac{1}{4}+\frac{1}{9} = \frac{13}{36}.$$

(g) $$\frac{.5\times.25-.25\times.0625}{.5-(.5)^2} = \frac{(.5)^3-(.25)^3}{.5-.25}$$
$$= (.5)^2+.5\times.25+(.25)^2 = .4375.$$

(h) $$\frac{2\times4\times8-.5\times.25}{5-1.5} = \frac{(4)^3-(.5)^3}{4-.5}$$
$$= (4)^2+4\times.5+(.5)^2 = 18.25.$$

177. Factor

(a) $(1+x)^2+(1+x^2)^2+2(1+x^2)+2x(1+x^2)$.
(b) $(x+y+z)^2+(x-y-z)^2+2x^2-2(y+z)^2$.
(c) $(1+2x+x^2)^2+(1-2x+x^2)^2+2(1-x^2)^2$.
(d) $(x\times y)^4-5(x^2+y^2)-10xy-24$.
(e) $p+q+r(p+q+1)+s(1-p-q)+r^2-s^2$.
(f) $x^2+y^2+x+y+2xy-xz-yz$.
(g) $p+q+r(p+q+r+s)-s(p+q+r+s)+r+s$.

(a) $(1+x)^2+(1+x^2)^2+2(1+x^2)+2x(1+x^2)$
$= (1+x)^2+(1+x^2)^2+2(1+x)(1+x^2)$
$= (2+x+x^2)^2$.

(b) $(x+y+z)^2+(x-y-z)^2+2x^2-2(y+z)^2$
$= (x+y+z)^2+(x-y-z)^2+2\left\{x^2-(y+z)^2\right\}$
$= (x+y+z+x+y-z)^2 = (2x)^2$.

(c) $(1+2x+x^2)^2+(1-2x+x^2)^2+2(1-x^2)^2$ is a perfect square, being $(2+2x^2)^2$.

(d) $(x+y)^4-5(x^2+y^2)-10xy-24$, by combining two middle terms the expression becomes $\left\{(x+y)^2-6\right\}\left\{(x+y)^2+1\right\}$.

(e) $p+q+r(p+q+1)+s(1-p-q)+r^2-s^2$
$= p+q+(r-s)(p+q)+r+s+r^2-s^2$
$= (p+q+r+s)(1+r-s)$.

(f) $x^2+y^2+x+y+2xy-xz-yz$
$= (x+y)^2+(x+y)-z(x-y$
$= (x+y)(x+y-z+1)$.

(g) $p+q+r(p+q+r+s)-s(p+q+r+s)+r+s$
$= p+q+(r-s)(p+q)+(r-s)(r+s)+r+s$
$(p+q+r+s)(1+r-s)$.

178. Shew that

$$(x+a)^3+(x-a)^3 = 2(x^3+3a^2x);$$

also that

$$(x+a)^3-(x-a)^3 = 2(3ax^2+a^3),$$

and from these formulæ simplify

(a) $(a+b+c)^3+(a+b-c)^3$;
(b) $(a+b+c)^3-(a-b-c)^3$;
(c) $(x+y+1)^3-(x+y-1)^3-2\left\{3(x+y)^2+1\right\}$.