or discharge section just up stream from the gate, as that is the problem investigated by Mr. Gibson. It is also assumed that the pipe line is of uniform thickness and diameter throughout its length. Then, during the period,

$$O<t \leqq 2L/a,$$

$$z = 1+F(t)/H_0,$$

where

$$F(t)/H_0 = m+\tfrac{1}{2}m^2[\phi(t)]^2-m\phi(t)\{1+m+\tfrac{1}{4}m^2[\phi(t)]^2\}^{\frac{1}{2}}.$$

and, during the period,

$$2L/a<t \leqq T,$$

$$z = 1+F(t)/H_0-F(t-2L/a)/H_0 = 1+F(t)/H_0-f(t)/H_0,$$

where

$$F(t)/H_0 = m-f(t)/H_0+\tfrac{1}{2}m^2[\phi(t)]^2- m\phi(t)\{1+m-2f(t)/H_0+\tfrac{1}{4}m^2[\phi(t)]^2\}^{\frac{1}{2}}.$$

For the linear law of gate movement, $\phi(t) = 1-t/T$, as already stated by Mr. Gibson.

These formulas applied to Mr. Gibson's first example give the following equations:—

For $O<t \leqq 0.35$ sec.,

$$F(t)/H_0 = 10.35008+53.56185\phi^2- 10.35008\,(11.35008\phi^2+26.78092\phi^4)^{\frac{1}{2}}$$

and, for $0.35<t \leqq 6.0$ sec.,

$$F(t)/H_0 = 10.35008-f(t)/H_0+53.56185\phi^2- 10.35008\{[11.35008-2f(t)/H_0]\phi^2+26.78092\phi^4\}^{\frac{1}{2}}.$$

As already noted by Mr. Gibson, the values of $F(t)/H_0$ and $f(t)/H_0$ are so small compared with some of the individual terms of the equations, that it is necessary to use logarithms in solving the equations.

Using the exact Alliévi equations as given previously, Table 1 has been prepared, showing the rise of pressure and also the differences between the values of the rise obtained by Alliévi equations and those obtained by Mr. Gibson's equations. The maximum divergence of less than one-half of 1% shows the remarkable agreement of the two methods. It is apparent, however, that the simplicity of the solution by the Alliévi formulas, without giving heed to magnitude and direction of waves in the computations, makes it far superior to that of Mr. Gibson as a working method.

The exact formulas of Alliévi also furnish equations similar to those given previously for the case of the opening of a valve at the lower end of a pipe line, either from fully closed position, or from some initial partial opening. As already stated, in their complete form they also give the pressure at any time for any point along the pipe line, thus covering the matter mentioned by Mr. Gibson as to be discussed by O. V. Kruse.* Of course, there are also exact equations for the velocity at any time for any point along the pipe line.

The partial differential equations for the general motion of water in pipes are based on the fundamental differential formulas for the motion of water in general. Unfortunately, these partial differential equations, four in number, cannot be integrated (not even by approximate arithmetic integration so far as the writer knows) without making the following simplifying approximations:—

1. Velocity in direction of axis of pipe considered uniform over any chosen section of the stream.
2. Skin friction and viscosity neglected.
3. Velocities at right angles to the axis of the pipe, due to expansion or contraction of the pipe by changes in pressure, neglected.
4. Pressure considered uniform over any chosen section of the stream.
5. Assumed that the pipe consists of individual circular elements independent of each other, which are freely extensible.
6. Assumed that the ratio of velocity of water in the pipe to the velocity of propagation of pressure changes is small enough, compared with unity, so that its addition thereto or subtraction therefrom can be neglected in every case at every instant.

By making these approximations, we obtain the so-called exact formulas of Alliévi.

*See *The Canadian Engineer*, October 9th, 1919, p. 370.

Mr. Gibson's method of taking account of skin friction (which is only approximate, as has been pointed out by William P. Creager), could easily be applied to the exact Alliévi formulas by changing the factor m so as to have it correspond at all times to (H_0+h_t) instead of to H_0.

The exact Alliévi formulas can be applied so as to take account of varying diameters and thicknesses of pipe in the same line, but they soon lead to so much complication that they become impracticable. In such cases, the writer uses the formulas as already given for a pipe of uniform diameter and thickness, but gives to m the value, $Q_0\Sigma(L/A)/gH_0\Sigma(L/a)$, where Q_0 equals the flow of water in a pipe at full gate-opening, L equals the length of any section of pipe, A, its cross-sectional area, and a, its individual value of the velocity of propagation of pressure changes. Also the factor, $2L/a$, must everywhere be changed to read $2\Sigma(L/a)$. This is confessedly an approximation made without mathematical proof, but it is probably exact enough for practical purposes in the majority of problems.

U. S. WATER POWER LEGISLATION

EFFORTS are being made by the engineers of the United States to obtain some measure of improvement in the water power legislation of that country. A circular letter has been drafted by the Water Conservation Committee of the U.S. Engineering Council and is being sent to every U.S. senator. The letter, which was signed by Alfred D. Flinn, secretary of the U.S. Engineering Council, is as follows:—

"Engineering Council asks your consideration of H.R. 3184, 66th Congress, 1st Session, which provides for the development of water powers in the navigable streams and on the public lands in the United States. This bill has passed the House of Representatives and is now before the Senate.

"During the past ten years, water power development in the United States on the public lands and in the navigable streams has languished. A certain small amount of development has taken place under conditions over which the Federal Government has no authority, but the best and most feasible development sites require Federal consent. Bills suitably providing for such consent have been under debate for a decade and the one above referred to (H.R. 3184) is approved by the heads of the several Federal departments concerned and also by the President. Advocates of legislation who have been on opposite sides of the water power question are generally agreed as to the merits of this bill and it is believed that it is the best measure that has yet been before Congress with any chance of passage. Probably, those who are now in disagreement after ten years of study and debate would be in the same position ten years hence. Therefore, it is bad public policy to wait longer; also, detrimental to the public interest and to industrial development.

"The importance of water power development in stimulating all kinds of industry, especially manufacturing, and the vital necessity of conserving and economizing our fuel supply, which, in turn, would largely relieve our congested transportation facilities, are so generally recognized that they need not be here emphasized.

"We, as engineers, are aware that the penalties that the country has already suffered by reason of the embargo on water power development, and would suffer from the consequences of further delay, are greater than those which could possibly be produced by all of the defects claimed against the bill by its few remaining opponents.

"We therefore hope you will lend your influence in favor of the speedy passage of the pending measure."

One can obtain a good conception of the remarkable growth of the American Association of Engineers during the last year when it is known that the number of paid employees at the national headquarters has grown in that time from four to sixty.