SOME MODERN METHODS OF ARCH CALCULATION.

WITH many comprehensive textbooks upon this subject, it is manifestly impossible to deal fully with it in a short space. The following abstracts from a paper read by Ewart S. Andrews, B.Sc., at a recent meeting of the Concrete Institute (Great Britain) are given in the hope that, as the author stated in concluding his remarks, a study of them would enable those who have not had the opportunity of a complete study of the arch theory in its more general aspects to follow the fundamental relations upon which scientific calculations are based and that their interest will be sufficiently aroused to encourage them to study the subject more fully.

The arch is a form of structure which possesses great advantages from the standpoints of beauty and economy, and from the earliest times the arch has been used in all kinds of constructional work. In the present paper arches are not considered at all from the point of view of architectural styles or orders, consideration being restricted to the calculation of the stresses in them.

The arch presents points of considerable difficulty from this point of view, and the resulting formulæ are elaborate. Some may contend that the formulæ are too elaborate, and that "simple practical rules" are just as good. One answer to this contention is that, such simple rules would be welcomed if they really were as good. The difficulty is that, unless simple rules are applicable over a wide range, and have been fully tested by scientific experiment, there is considerable danger in their use.

There is, unfortunately, among some practical engineers a strong antipathy to complicated formulæ—an antipathy which is usually stronger in proportion as the formulæ are not understood—but closer acquaintance with such formulæ and some useful spade work in the form

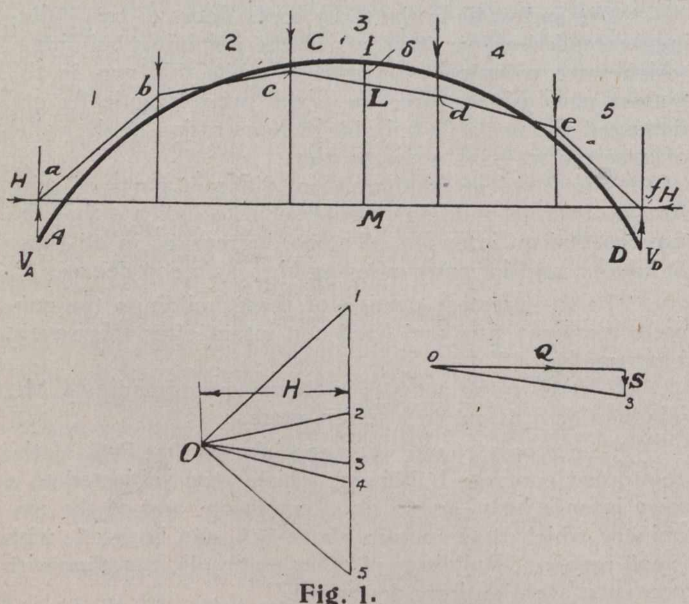

Fig. 1.

of the compilation of tables and diagrams usually helps to dispel much of the dread. The primary reason of the difficulty in the determination of the stresses in arches arises from the fact that in most cases the arch is what is called "a statically indeterminate structure," so that the forces acting upon the arch cannot be found by the ordinary laws of statics. Exactly similar difficulties arise in the case of stiffened suspension bridges, continuous beams, and slabs.

Determinations of Reactions in an Arch.—The stresses in an arch can be found as soon as we can find the magnitude and position of the reactions. These reactions R may be considered as compounded of vertical components V and horizontal thrusts H. If, as is usual, all the loads on the arch are vertical, the horizontal thrust H must be the same at each end. We can then draw the line of pressure, linear arch or equilibrium polygon (three alternative names for the same thing) for the given load system upon the arch.

Then, by Eddy's theorem that "the bending moment at any point of an arch is equal to the product of the hori-

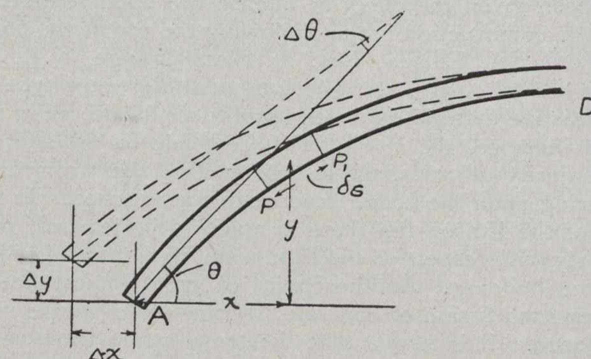

Fig. 2.

zontal thrust into the vertical intercept between the centre line of the arch and the line of pressure," we have—

Bending moment at any point $P = H\,\delta$

If the line of pressure comes below the centre line of the arch, the upper surface or extrados of the arch will tend to become in tension, and if the line of pressure is above the centre line of the arch, the lower surface, or intrados, of the arch tends to become in tension.

Stresses in the Arch.—To obtain the stresses in the arch we first find the thrust or normal pressure Q, and the shearing force S, at the point by resolving the thrust O_3 at the point under consideration along and perpendicular to the centre line of the arch at the given point.

Then, if A is the area of the section, and M_o, M_t the compression and tension moduli, we have—

$$\text{Maximum tension stress} = t = \frac{H\,\delta}{M_t} - \frac{Q}{A} \quad . \ (1)$$

$$\text{Maximum compression stress} = c = \frac{H\,\delta}{M} - \frac{Q}{A} \quad . \ (2)$$

$$\text{Mean shear stress} = s = \frac{S}{A} \quad . \ . \ (3)$$

There is therefore no difficulty whatever in the calculation of stresses in arches when once we know the horizontal thrust.

Cases in which the horizontal thrust can be found without the elastic theory of arches are as follows:—

(1) Parabolic arch uniformly loaded over whole span—

$$H = \frac{wl^2}{8v}$$

w = load per unit length, l = span of arch, v = rise of arch

(2) Parabolic arch with uniform load extending from an abutment to the centre—

$$H = \frac{wl^2}{16v}$$

(3) Arch of any shape provided with hinges.—In this case the line of pressure can be drawn by the well-known graphical construction for making a link-polygon pass through any three given points; then the polar distance of the vector diagram gives the horizontal thrust desired.