$$b + b_1 y_1 + b_2 y_1^{2} + \dots\dots + b_{r-1} y_1^{r-1},$$

where b, b_1, &c., are clear of the surd y_1. The corresponding coefficient in $F_2(x)$ is

$$b + b_1 z y_1 + b_2 z^2 y_1^{2} + \&c.$$

Therefore, $b_1(z-1)y_1 + b_2(z^2-1)y_1^{2} + \&c. = 0$.

Since the surds present in this equation are surds occurring in $f(p)$, and $f(p)$ is in a simple form, the coefficients, $b_1(z-1)$, $b_2(z^2-1)$, &c., must (Cor. 1. Def. 9) vanish separately. But, since z is an r^{th} root of unity, distinct from unity, r being a prime number, none of the expressions, $z-1$, z^2-1, &c., vanish. Therefore b_1, b_2, &c., must all be zero: which is inconsistent with the assumption that the surd y_1 is present in the coefficient selected. Therefore $F_1(x)$ is not equal to $F_2(x)$; and we proved that it has no common measure with $F_2(x)$. Therefore no term in (1) is equal to a term in (2); and all the terms, ϕ_1, ϕ_2, ..., ϕ_{2n}, are unequal. In the same way it appears that all the terms, ϕ_1, ϕ_2, ..., ϕ_{nr}, are unequal.

The terms, ϕ_1, ϕ_2, ..., ϕ_{nr}, thus proved unequal, are the unequal cognate functions of $f(p)$, obtained by giving definite values to the surds in F, [which, from the manner in which F was generated, are necessarily surds occurring in $f(p)$], and framing the cognate functions without reference to the surd character of these surds. For, in framing the cognate functions, ϕ_1, ϕ_2, ..., ϕ_{nr}, all the surds in $F_1(x)$, except y_1, were considered as definite; and no numerical multipliers (such as z_1, z_2, &c., in Def. 6) were affixed to them. If F contained all the surds in $F_1(x)$, except y_1, our point would be easily established. It may happen, however, that F does not contain all the surds in $F_1(x)$ except y_1. Other surds may have disappeared from it, along with y_1. Let t be one of these, if there be such: and let its index be $\frac{1}{s}$. Then, in virtue of the s values that may be given to t, the cognate functions of $f(p)$, taken on a non-recognition of the surd character of those surds alone which appear in F, must include s groups of such terms as

$$\phi_1, \phi_2, \dots\dots\dots \phi_{nr}, \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

In general, if t, t_1, &c., be the surds in $F_1(x)$, besides y_1, which are not in F; and if $\frac{1}{s}$, $\frac{1}{s_1}$, &c., be the indices of the surds t, t_1, &c.,