When $x = \sqrt{a^2 + b^2}$, an hypotenuse is to be drawn on a and b put at right angle, because, if both members of the equation are raised to the second power, we obtain

$$x^2 = a^2 + b^2$$

The fifth formula is proved in the same way.

N.B.—The student is supposed to know how to draw a fourth proportional to three given lines, a mean proportional between two given lines, etc.

IV.

Let it be illustrated by a few examples, that any expression may easily be reduced to one of the five preceding formulas.

Ex. 1. Let $x = \frac{a\,b\,c}{r\,s}$

then $x = \frac{a\,b}{r} \times \frac{c}{s}$ by decomposing.

Let the *fourth proportional* expressed by $\frac{a\,b}{r}$ be drawn and represented by y, then

$$x = y \times \frac{c}{s} = \frac{y\,c}{s} \text{ (2nd formula).}$$

Ex. 2. Let $x = \frac{a^3 + b^2\,c + d\,h\,m}{p^2 + q^2}$

Let an hypotenuse be drawn on p and q, and represented by y, then

$$x = \frac{a^3 + b^2\,c + d\,h\,m}{y^2} =$$

$$= \frac{a^3}{y^2} + \frac{b^2\,c}{y^2} + \frac{dhm}{y^2} = \frac{a\,a\,a}{y\,y} + \frac{b\,b\,c}{y\,y} + \frac{d\,h\,m}{y\,y}$$

Let each of these last three terms be reduced as in the Ex. 1, and the three lines found be represented respectively by u, v, z, then

$$x = u + v + z \quad \text{(1st formula).}$$

Ex. 3. Let $x = \sqrt{3\,a^2}$

then $x = \sqrt{3\,a \times a}$ (3rd formula).