Mathematics.

All communications intended for this column should be sent before the 20th of each month to C. Clarkson, B.A., Seaforth, Ont.

THIRD CLASS ARITHMETIC 1887.

1. PROVE the rule for the multiplication of fractions.

The two fundamental operations on fractions are,

(1.) To multiply a fraction by a whole number.

(2.) To divide a fraction by a whole number. We take these for granted and put the proof in the form of an arithmetical equation.

Suppose $\frac{6}{7}$ and $\frac{5}{11}$ are the given fractions of which the product is required. Then product $=\frac{6}{7}\times\frac{5}{11}$

$\therefore$ 11 times product $=\frac{6}{7}\times\frac{5}{11}\times 11$. But $\frac{5}{11}\times 11=5$, from (1)

$\therefore$ 11 times product $=\frac{6}{7}\times 5$, and this $=\frac{30}{7}$, from (1)

$\therefore$ product $=\frac{30}{7}\div 11=\frac{30}{77}$, from (2)

and we see that the produet $=\dfrac{6\times 5}{7\times 11}$, hence the rule.

N.B.—In some proofs given by standard authors the explanation of the rule is needlessly abstract, so that the explanation itself is more difficult of apprehension than the fact to be explained. In other cases the attempted proof is in reality only an example of the sophism *petitio principii.*

Simplify $\dfrac{(7\frac{1}{4}-3\frac{1}{2})\times\{4\frac{1}{5}-(2\frac{1}{3}-1\frac{7}{10})\}}{(7\frac{1}{4}+3+\frac{1}{2})\div(1\frac{1}{2}-9\frac{1}{2}\times\frac{9}{77})}$

Multiply N and D by 4, and remove the inner brackets thus :—

$$\frac{(29-14)\times(4\frac{2}{10}-2\frac{1}{3}+1\frac{7}{10})}{(29+14)\div(1\frac{1}{2}-9\frac{1}{2}\times\frac{9}{77})}$$

$$=\frac{15\times(5\frac{9}{10}-2\frac{1}{3})}{43\div(1\frac{1}{2}-1\frac{17}{154})}=\frac{15\times\frac{107}{30}}{43\div\frac{60}{154}}$$

$$=\frac{107\times 30}{2\times 43\times 77}=\frac{1605}{3311}.$$

It may be well to remark that examiners in Ontario have never been in the habit of giving more than one half the full marks for the bare answer without a full exhibition of the method of solution. The chief value of such a question as this is a disciplinary value. If the result is obtained in a disorderly, slip-shod way, the whole benefit of such exercises is lost, and the student's time is somewhat worse than wasted by confirming him in habits of carelessness and haste.

(2.) A, B and C rent a pasture for \$92. A puts in 6 horses for 8 weeks ; B 12 oxen for 10 weeks ; C 50 cows for 12 weeks. If 5 cows are reckoned as 3 oxen, and 4 oxen as 3 horses, what shall each pay ?

SOLUTION.—5 cows = 3 oxen, $\therefore$ 600 cows = 360 oxen,

3 horses = 4 oxen, $\therefore$ 48 horses = 64 oxen.

Now, 6 horses for 8 weeks = 48 horses for one week

= 64 oxen for 1 week

Also, 12 oxen for 10 weeks = 120 oxen for 1 week.

And 50 cows for 12 weeks = 600 cows for 1 week

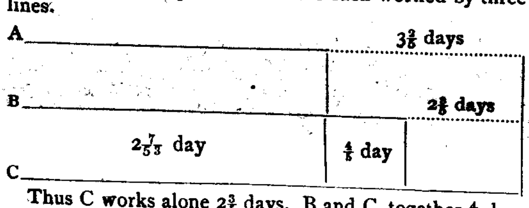
= 360 oxen for 1 week.

Thus the shares are as 64 : 120 : 360, that is

as 8 : 15 : 45. Therefore A's share is $\frac{8}{68}$ or $\frac{2}{17}$ of \$92 = \$$10\frac{14}{17}$, which leaves \$$81\frac{3}{17}$ to be paid by B and C. whose shares are as 1 : 3, hence B must pay $\frac{1}{4}$ and C $\frac{3}{4}$ of \$$81\frac{3}{17}$, or B pays \$$20\frac{5}{17}$ and C three times as much = \$$60\frac{15}{17}$.

3. A does a work in ten days ; B in 9 days ; C in 12 days. All begin together, but A leaves $3\frac{2}{5}$ days before the completion ; B in $2\frac{3}{5}$ days before the completion. In what time was the work done ?

SOLUTION.—Represent the time each worked by three lines.



Thus C works alone $2\frac{3}{5}$ days, B and C together $\frac{4}{5}$ day, and A, B and D together for the remainder of the whole time.

Thus C does $2\frac{3}{5}\times\frac{1}{12}=\frac{13}{60}$ of the work alone

B and C do $\frac{4}{5}\times(\frac{1}{12}+\frac{1}{9})=\frac{7}{45}$ of the work, leaving $\frac{180}{180}-(\frac{13}{60}+\frac{7}{45})$ or $\frac{113}{180}$ to be done by A, B and C together, at the rate of $(\frac{1}{10}+\frac{1}{9}+\frac{1}{12})$ per day $=\frac{53}{180}$ per day. For this they require $(113\div 53)$ days or $2\frac{7}{53}$ days. Hence the whole time $=2\frac{7}{53}+3\frac{2}{5}=5\frac{141}{265}$ days.

N.B.—The algebraical solution is more simple and straightforward. Thus let x = whole time by C, etc.

4. Prove the rule for division of decimals.

Suppose we have to divide .0083 by .854.

Quotient $=\dfrac{.0083}{.854}=\dfrac{.0083\times 1000}{.854\times 1000}=\dfrac{8.3}{854}$, which reduces the question to the elementary stage, viz., dividing a decimal by a whole number, and establishes the rule.

Divide to six decimal places .0078539 by .9921461. We shall use the contracted method with the combined method of multiplication and subtraction :—OPERATION.

992,1,4,6,1) 78539 (0.007916
9089
160
61
7

5. On March 23rd a bank gives me \$845 for a note of \$860. When is the note due, interest 8 % ?

Bank Int = \$860 − \$845 = \$15

Int. on \$860 for 1 day $=(860\times\frac{8}{100})\div 360=\$\frac{43}{225}$

$\therefore$ No. days $=15\div\frac{43}{225}=78\frac{21}{43}$ or 79 days.

Subtract 3 days grace, 76 days, from March 23rd gives June 7th.

6. Find the cost in sterling, of 184 tons, 17 cwt, 3 qrs, 14 lbs of copper, invoiced to a Toronto importer at £87, 17s. 11d. per ton.

Price of 1 ton = £87 17 11

" " 1 cwt = £ 4 7 $10\frac{3}{4}$

" " 1 qr = £ 1 1 $11\frac{1}{16}$. Hence we have

£87 17 11 × 184 =	£16172 16 8	
4 7 $10\frac{3}{4}$ × 17 =	74 14 $2\frac{3}{4}$	
1 1 $11\frac{1}{16}$ × $3\frac{1}{2}$ =	3 16 $10\frac{22}{32}$	
Total price =	£16251 7 $9\frac{21}{32}$.	

7. I bought certain 4 per cent. stock at 75 and after a number of years sold out at 95, and found that I had made $7\frac{1}{2}$ % per annum, simple interest. How long did I hold the stock ?

SOLUTION.—\$75 invested brings \$4 a year

\$75 sold out " \$20 profit

\$75 at $7\frac{1}{2}$ % " \$$\frac{45}{8}$ per annum.

Thus we get the equation

(\$4 × No. years) + \$20 = \$$\frac{45}{8}$ × No, years

or \$20 = \$$\frac{13}{8}$ × No. years

i.e. $20\div\frac{13}{8}$ = No. years = $12\frac{4}{13}$.

VERIFICATION.

\$4 a year for $12\frac{4}{13}$ years yields \$$49\frac{3}{13}$ interest.

\$20

\$75 at $7\frac{1}{2}$% for $12\frac{4}{13}$ years gives \$$69\frac{3}{13}$ profit.

8. There is a mixture of vinegar and water in the proportion of 93 parts vinegar to 7 parts water. How much water must be added so that in 25 parts of the mixture there may be 2 parts water ?

A certain number must be added. Then we have this result :—

93 : 7 + Reqd No. = 23 : 2

or 2 × 93 = 23 (7 + Reqd No.)

i.e., 186 = 161 + 23 times Reqd No.

or 25 gallons = 23 times Reqd No.

$\therefore 1\frac{2}{23}$ gals = Reqd No. to be added.

OTHERWISE :—Water now $=\frac{7}{93}$ of the vinegar.

But it must be made $=\frac{2}{23}$ of the vinegar ;

Amount to be added $=(\frac{2}{23}-\frac{7}{93}$ of vinegar

$=\frac{25}{23}\times\frac{93}{93}$ gals $=1\frac{2}{23}$ gals, as before.

9. I invested \$10,000, but sold out at 20 % discount. How much must I borrow at 4 % so that by investing all at 8 % I may just retrieve my loss ?

N.B.—The statement of this question is enigmatical and unsatisfactory. What do the examiners mean by the phrase "I borrow money at 4 % ?" Is it at 4 % per annum ? Is it at simple interest or compound interest ? When is the loss to be retrieved, immediately or at the end of a year ? If they mean that the difference between the present worth of the money borrowed at 4 % and the present worth of the whole loaned at 8 % = the \$2,000 lost by selling out \$10,000 at 20 % discount, they might have taken the trouble to say as much. It is to be hoped that no such indefinite question will be found on the arithmetic paper next July. We can surely have questions of sufficient difficulty without resorting to vague enigmas like this. It would be like taking a ticket for a lottery to guess at the meaning intended. Perhaps the words "at par" are intended to be understood after \$10,00c, and "per annum" after 4 %, and 8 %, but who can tell ?

10. A square field containing $27\frac{1}{2}$ acres has a diagonal path across it. What is the length of the path in yards ?

2 (side)^2 = (diagonal)^2. Euc. I. 47.

$\therefore$ diagonal = side × $\sqrt{2}$. Now side = $\sqrt{\text{area}}$, or

side $=\sqrt{(\frac{55}{2}\times 4840 \text{ sq. yds.})}$

$=\sqrt{(55\times 2420)}$

$=\sqrt{(55^2\times 2^2\times 11)}$

$=110\sqrt{11}$; $\therefore$ side $\sqrt{2}=110\sqrt{11}\times\sqrt{2}=110\sqrt{22}$ yds.

11. When the temperature of a cube of zinc is raised from 32° F. to 212° F. each dimension is thereby increased 3%. Find the percentage increase in the bulk.

$3\% = \frac{3}{10}\div 100 = \frac{3}{1000}$. So that if the side is 1000 at first, it becomes 1003 after expansion. Consequently the cubical content increases from \$$1000^3$ to 1003^3.

Now $1003^3=(1000+3)^3=1000^3+3^3+9000\times 1003$

And $1000^3=1000^3$

Increase of bulk $=3^3+9000\times 1003=9027027$

That is on

1,000,000,000 the expansion is 9027027

$\therefore$ on 100 " " " ·9027027

12. Water is flowing at the rate of 10 miles per hour through a pipe 14 inches in diameter, into a rectangular reservoir 187 yds by 96 yds. In what time will the surface be raised one inch.

One inch deep on the reservoir will contain $187\times 96\times 9\times 144$ cubic inches.

10 miles per hour $=5280\times 12\times 10$ inches in 60 minutes $=5280\times 2$ inches per min.

Area of section of pipe $=\frac{22}{7}\times 7^2=7\times 22$ square inches,

Hence $7\times 22\times 5280\times 2$ cub. in. are brought in per min.

No. min. reqd $=\dfrac{187\times 96\times 9\times 144}{7\times 22\times 5280\times 2}$

$=\dfrac{17\times 9\times 36}{7\times 55}=14.3064$ minutes.

CORRESPONDENCE.

The following problem by MR. WM. LINTON, NEW HAMBURG was mislaid.

A township issues debentures with coupons attached to borrow \$2000 at 6 %. The tender of C at \$2040 is accepted. He receives one fourth of the principal back each year, *i.e.*, \$500, and the interest on the unpaid principal, each year, viz \$120, \$90, \$60 and \$30. What rate % does C receive on his investment ?

This is an interesting practical question to which we hope to receive a number of solutions.

We have a neat solution of the cistern question given in the March number. It is by MISS ANNA TAIT (aged 14). a pupil of MR. PALLES, Carrying-Place. To the same question solutions were sent too late for last issue by MR. FLANAGAN, Iroquois. MR. J. B. MORRISON, Birtle, Man. and MR. T. F. FLAHERTY, Lucan. The last named gentleman has examined the "bankrupt" of the January No. and pronounces him "without doubt a fraud." He agrees with MISS GERRIE that there can be no \$20,000 assets such as are spoken of in the problem, and demonstrates this by assuming for the sake of argument that his total assets were just \$20,000. We do not remember where this problem came from, but if it was not originally intended for an impossible question, perhaps there may have been some figures misplaced, thus spoiling the question for solution.

A. B. M. asks which of the following statements is the better to place before a junior class :—

(*a*) 1 acre costs \$85 ; $\therefore$ 5 acres cost 5 × \$85

(*b*) 1 " " \$85 ; $\therefore$ 5 " " \$85 × 5 The latter is the more correct form, unless we understand \$(5 × 85) and \$(85 × 5), in which case both are accurate. For we can say :—If 1 acre cost \$1, 5 acres cost \$5 $\therefore$ at \$85 per ac, the cost would be \$5 × 85. Usually, however, we say \$85 multiplied by 5, so as to avoid the appearance of making the multiplier a concrete number. In the case of areas this needs careful explanation :—Thus 4 ft. × 3 ft. = 12 sq ft. is nonsense, and is liable to produce injurious confusion of thought in the mind of the pupil. Some of our readers remember an arithmetic in our schools which taught that 2s. 6d. × 2s. 6d. = 6s. 3d. But the former times were not better than these. That was thirty years ago.

CORRECTION,—Question 16, April 1st, should read :—Prove that $a^3+b^3+c^3-3abc=(a+b+c)(a+bx+cx^2)(a+bx^2+cx)$, where $x=\frac{1}{2}(-1+\sqrt{-3})$. Several solutions received to No. 17 all agree in "begging the question." No good solution has come to hand.

G. E. T., *Lefroy*. "In the expression $\frac{5}{6}+\frac{3}{4}\div\frac{7}{8}\times\frac{7}{9}+\frac{2}{3}$ of $1\frac{1}{2}$, what signs must be dealt with first, secondly, etc ?"

ANSWER. By a general convention the expressiou is understood to mean the sum of three terms thus :—$\frac{5}{6}+[\frac{3}{4}\div(\frac{7}{8}\times\frac{7}{9})]+(\frac{2}{3}$ of $1\frac{1}{2})$; so that the signs ×, ÷, and "of" must be understood to operate before the signs +, +. It would eliminate all ambiguity to use brackets.

J. MCL., *Lynn Valley*. "A man assisted part of the time by a boy completed a piece of work in 15 hours. The man got for his share of the pay five times as much as the boy received for his share ; but the man was paid twice as much as the boy in proportion to the amount of work done by each. In how many hours could the man do the work without assistance ?"

SOLUTION.—The man worked 15 hours. Suppose he did $\frac{1}{x}$ of the whole work, and the boy $\frac{1}{y}$ of the same per hour. Also suppose the boy worked z hours. Then the man's work is represented by $\frac{15}{x}$, and the boy's part by $\frac{z}{y}$. Consequently their wages are represented by $\frac{30}{x}$ and $\frac{z}{y}$. Thus we have the proportion

$\frac{30}{x}:\frac{z}{y}=5:1$; or $\frac{30}{x}=\frac{5z}{y}$ *i.e.* $\frac{6}{x}=\frac{z}{y}$; which, being interpreted, means that the boys part of the job was as much as the man could have done in 6 hrs. Therefore *alone* he would require 15 + 6 = 21 hrs. ANS.