$$\frac{\epsilon c_1}{T_0} = \frac{\epsilon h_1}{T^2} \qquad (27)$$

If we introduce $z = y - \epsilon h_1$, therefore $\frac{dz}{dt} = \frac{dy}{dt}$;

$$\frac{d^2z}{dt^2} = \frac{d^2y}{dt^2} \quad - \quad - \qquad (28)$$

from (26) follows a differential equation of the second order

$$\frac{d^2y}{dt^2} + \frac{1}{T_0} \cdot \frac{dy}{dt} + \frac{y}{T^2} = 0 \quad - \qquad (29)$$

This is a linear differential equation of the second order with constant coefficients. For the solution we say:

$$r^2 + \frac{1}{T_0} r + \frac{1}{T^2} = 0$$

The solution of this quadratic equation will give us the values of the differential equation:

$$r = -\frac{1}{2T_0} \pm \sqrt{\frac{1}{(2T_0)^2} - \frac{1}{T^2}}$$

We know that according to the theory of the linear differential equations three solutions may be found.

If the radical is positive or zero, the solution of the differential equation represents a non-periodical function, that is:

$$y = R_1 . e^{-\frac{t}{2T_0}} + R_2 t e^{-\frac{t}{2T_0}}$$

$$\text{if } \frac{1}{T_1^2} = \frac{1}{T^2} - \frac{1}{(2T_0)^2} = 0 \qquad (30a)$$

$$y = (R_1 e^{+\frac{t}{T_1}} + R_2 e^{-\frac{t}{T_1}}) e^{-\frac{t}{2T_0}}$$

$$\text{if } \frac{1}{T_1^2} = \frac{1}{T^2} - \frac{1}{(2T_0)^2} = \text{negative} \qquad (30b)$$

But,

$$y = R e^{-\frac{t}{2T_0}} . \sin(\beta + t/T_1)$$

$$\text{if } \frac{1}{T_1^2} = \frac{1}{T^2} - \frac{1}{(2T_0)^2} = \text{positive} \qquad (30c)$$

represents a damped harmonic.

These three conditions, due to the different values of the radical of the quadratic equation can be written also, substituting for T and T_0 the values of equations 24:

$\frac{1}{T_1^2}$ becomes zero if $T = 2T_0$ or $\frac{A}{a} = \frac{4L}{n^2 g}$ (31a)

$\frac{1}{T_1^2}$ becomes negative if $T > 2T_0$ or $\frac{A}{a} > \frac{4L}{n^2 g}$ (31b)

$\frac{1}{T_1^2}$ becomes positive if $T < 2T_0$ or $\frac{A}{a} < \frac{4L}{n^2 g}$ (31c)

As mentioned before, equations 30a and 30b, with the conditions of 31a and 31b, represent non-periodical function, that is:

Form 1 is the expression for a damped oscillation;

Forms 2 and 3 represent non-periodical movements, i.e., a transition from one quiescent level to another without any oscillations.

As will be seen later from an example, n has in most cases a value which lies between 2 and 1 seconds; therefore, the condition for a non-periodic water level fluctuation is:

$\frac{A}{a}$ is equal to or greater than $\frac{L}{35}$.

Now, for $L = x$ miles

A should be equal to or greater than $150.x.a$.

This case may well occur when a pond is used as the surge tank. With artificially constructed surge tanks $\frac{A}{a}$ is considerably smaller; the following investigations are therefore limited to the first form of oscillations. As $z = y - \epsilon h_1$, equation (30) may be written

$$z = -\epsilon h_1 + R e^{-\frac{t}{2T_0}} \sin(\beta + t/T_1) \quad - \qquad (32)$$

and the differentiation with respect to t gives (since $\frac{dz}{dt} = s$)

$$s = R e^{-\frac{t}{2T_0}} \frac{1}{2T_0} \left\{ \frac{2T_0}{T_1} \cos(\beta + t/T_1) - \sin(\beta + t/T_1) \right\} \qquad (33)$$

If we substitute $tg\gamma = \frac{2T_0}{T_1}$ and consider that

$$\frac{1}{T_1} = \frac{1}{T^2} - \frac{1}{4T_0^2} \quad - \quad - \qquad (34)$$

we get:

$$s = \frac{R}{T} . e^{-\frac{t}{2T_0}} \sin(\gamma - \beta - t/T_1) \qquad (35)$$

and the integration constants R and β are determined from the initial conditions, i.e., from the location and the condition of movement of the water level in the surge tank at the time $t = 0$.

It will be noted from the description of the phenomena for that case that for the shut-down and in the very moment of it, the water surface is at the distance h_1 under the elevation $n - n$. Therefore, for $t = 0$, $z = z_0 = -h_1$. The initial value for $s = s_0$ at the time $t = 0$, must be assumed as

$$s_0 = \frac{Q_1 - \epsilon Q_1}{A} = (1 - \epsilon) c_1 \quad - \qquad (36)$$

This assumption does not only consider a sudden shut-down, but assumes also a sudden beginning of a