6. $\text{Exprs}=\frac{x^2+1}{x^2-1}\cdot\frac{x^3-1}{x^6+1}\cdot\frac{(x^2-1)^2+x^2}{x^4+x^2+1}$.

$=\frac{x^2+1}{x^2-1}\cdot\frac{x^3-1}{x^6+1}\cdot\frac{x^4-x^2+1}{x^4+x^2+1}$.

Now, $x^4-x^2+1=\frac{x^6+1}{x^2+1}$ and $x^4+x^2+1=(x^2+x+1)(x^2-x+1)$.

$\therefore \text{Exprs}=\frac{(x^2+1)(x^2+x+1)(x-1)(x^6+1)}{(x+1)(x-1)(x^6+1)(x^2+1)(x^2+x+1)(x^2-x+1)}=\frac{1}{x^3+1}$

7. (a) $s=(a+l)\frac{n}{2}$. $a+d=4$, $a+4d=8\frac{1}{2}$, $\therefore d=\frac{3}{2}$, $\therefore a=\frac{5}{2}$,

$\therefore s=(\frac{5}{2}+\frac{17}{2})\frac{5}{2}=27\frac{1}{2}$.

(b) $ar^2=3$, and $ar^4=27$, $\therefore r=\pm 3$, $\therefore a=\frac{1}{3}$, and the series is $\frac{1}{3}+1+3+9+27=40\frac{1}{3}$; or, taking the negative sign, $\frac{1}{3}-1+3-9+27=20\frac{1}{3}$.

8. 1 man+1 woman+2 boys+1 girl get 100*s*.
i.e. (2 boys+1 girl)+(2 boys—1 girl)+2 boys+1 girl get 100*s*.
or, 6 boys and 1 girl would get 100*s*. (*A*)
Again, 1 man+1 girl got 50*s*.
i.e. (2 boys+1 girl)+1 girl got 50*s*.
or, 1 boy and 1 girl get 25*s*. (*B*)
Subtracting *B* from *A*, 5 boys get 75*s*.; and the shares are—each boy 15, the girl 10, the woman 20, and the man 40 shillings.

9. Observe that in *A* and *B* the sum of the coeff. in the odd places, signs included=sum of coeff. in even places=140, hence each is divisible by $x+1$. In *C* the sum of all the coeff. vanishes, hence $x-1$ is a factor. See McLellan's Teachers' Handbook—the "K Method," p. 92. Thus for *A* we have

```
   |1+14+67+126|+72
 -1| — 1-13- 54|-72
   |1+13+54+|72
 -3| - 3-30-|72
   |1+10+24=(x+8)(x+3)
```

$\therefore A=(x+1)(x+3)(x+3)(x+8)$, and similarly,
$B=(x+1)(x+3)(x+5)(x-6)$.
$C=(x-1)(x+3)(x+5)(x+6)$.

$\therefore$ G.C.M. $=x+3$. See also McLellan's Handbook, p. 105, for another method.

10. Let r=rent in pounds, w=water-rate per £, p=poor-rate do.
Then $r+rw+rp=3000s$.
$\frac{9}{10}r+\frac{9}{10}r(\frac{19}{20}w)+\frac{9}{10}r(\frac{3}{4}p)=2601s$.
$r+r(\frac{19}{20}w)+r(2p)=3390s$.
Simplifying we have
$r(1+w+p)=£150$.
$r(20+19w+15p)=2890$.
$r(20+19w+40p)=3390$.
From 2nd & 3rd, $rp=£20$. Then from 1st and 2nd, $rw=£10$, and $r=£120$.

GEOMETRY.

1. If the square described on one side of a triangle be equal to the sum of the squares described on the other two sides, prove that the angle contained by these sides is a right angle.

2. Prove that the quadrilateral formed by joining the middle points of the sides of any quadrilateral is a parallelogram; and that its area is half that of the given quadrilateral.

3. If a right line be divided into two equal and into two unequal parts, prove that the sum of the squares described on the unequal parts is double the square on half the line and double the square on the intermediate part.

4. Divide a given right line into two parts so that the sum of their squares shall be equal to a given area. Show how the requisite construction is made, and state when it is impossible.

5. If a quadrilateral be inscribed in a circle prove that the sum of one pair of its opposite angles is equal to the sum of the other pair.

6. If a quadrilateral be circumscribed to a circle prove that the sum of one pair of its opposite sides is equal to the sum of the other pair.

7. Find the locus of the centre of a circle whose circumference passes through two given points.

8. Describe a circle through two given points and touching a given circle. Determine the number of solutions of the problem, and when it is impossible.

9. Prove that the lines drawn bisecting the three internal angles of a triangle pass through a common point: and show that the same theorem holds for the lines bisecting one internal and two external angles of the triangle.

10. Divide the circumference of a circle into thirty equal parts, giving the requisite construction.

Hints and Solutions.

1. I. 48.

2. Let *ABCD* be the quadrilateral, *E* the mid. pt. of *AB*, *F* of *BC*, *G* of *CD*, *H* of *DA*. Draw the diagonal *DB*. Then in the $\triangle ADB$ the sides are cut proportionally, $\therefore HE$ is par. to *DB*. (VI.2)
So also *GF* is parallel to *DB*, $\therefore HE$ is parallel to *GF*, (I. 30)
So " *HG* " *EF*, $\therefore HGFE$ is a ▱m.
Also *AD* is double *AH*, $\therefore \triangle ADB$ is quadruple $\triangle AHE$. (VI.19)
i.e. $AHE=\frac{1}{4}ADB$; so also $FCG=\frac{1}{4}DBC$, and thus
AHE and $FCG=\frac{1}{4}$ whole quad. In the same way *EBF* and *DHG* $=\frac{1}{4}$ quad., i.e. these 8 $\triangle$'s$=\frac{1}{2}$ whole quad. $\therefore$ the remainder, viz., the ▱m $HF=\frac{1}{2}$ quad.

3. Let *A* C D *B* be the given line,
Then (II. 7) $BC^2+CD^2=2BC\cdot CD+DB^2$,
or $AC^2+CD^2=2AC\cdot CD+DB^2$. $\therefore AC=BC$.
Also (II. 4) $AC^2+CD^2+2AC\cdot CD=AD^2$.
Add these equals and take $2AC\cdot CD$ from each side,
and $2AC^2+2CD^2=AD^2+DB^2$.

4. Let *AB* be the given line, *S* the side of a square=given area, (by II. 14). At *B* draw *BK* making $\frac{1}{2}$ rt. ∠ with *AB*. From centre *A* with radius=*S* describe a circle cutting *BK* in *P*. From *P* drop perp. *PQ* on *AB*. *Q* is the req. pt. The proof is obvious. *S* can never be $>AB$ (I. 20), and even when $S=AB$, *Q* coincides with one end of the line.

5. III. 22. See H. Smith's Geom., p. 177, for neat proof.

6. The sides are tangents to the circle. The tangents drawn from the same pt. are equal. Hence by addition the result req.

7. Let *AB* be the strt. line joining the given pts. Bisect *AB* at rt. ∠'s by the strt. line *CD*. The centre must lie in *CD*. (III. 3.)

8. Let *A*, *B*, be the given. pts. Take any pt. *C* in the given circum. Describe a circle thro. *A*, *B*, *C*, (IV. 5). If this circle does not touch the given circle, let *D* be the other pt. of intersection. Produce *AB* and *CD* to meet at *E*. Draw *EF* touching given circle at F. Describe a circle thro. *A*, *B*, *F*, this will be the circle req. (III. 35) and (III. 37). Two strt. lines can be drawn from *E* touching given circle, $\therefore$ two solutions possible. The construction fails when *AB* is parallel to *CD*. In that case draw strt. line parallel to *AB* touching the circle in *K*. Describe a circle round *ABK*. *AB* may also lie within the given circle. If it be equal to its diameter the solution fails.

9. Let *ABC* be the triangle. Bisect the angles at *B* and *C*. Let the lines meet at *G*. Join *AG* and produce it to meet *BC* in *H*. Then (VI. 3), $AB.BH=AG\cdot GH$, and $AC\cdot CH=AG\cdot GH$ $\therefore$ (V.11), $AB:BH=AC:CH$, and $\therefore$ (V. 16), $AB:AC=BH:CH$, and $\therefore$ (VI. 3) *AH* bisects the angle at *A*. In the case of external angles we shall merely require to use (VI. A.).

10. Inscribe a regular hexagon (IV. 15), and let *AB* be one side. Also inscribe a regular pentagon (IV. 14), one of whose sides is *AC*. Then of 30 equal parts of the circum. *AB* cuts off 5, and *AC* cuts off 6, $\therefore BC$, their diff., contains one. Join *BC* and place 29 other lines round the circle each=*BC*.

Problems for Solution.

By J. H. Thomson, Monkton, Ont.—

1. The current in the centre of a stream is twice as great as it is at the edge. A man rows up the edge in 30m., and down the centre in 20m.; find how long it would take him to row up the centre of the stream.

2. I have two silver coins one of which is $\frac{9}{10}$ fine, and its diameter is 1 inch; the other one is $\frac{8}{10}$ fine, and its diameter is $1\frac{1}{4}$ in.; the first one is worth $\frac{1}{3}$ as much again as the second; find of ratio their thicknesses.

Algebra.

3. Solve the equations—

(1) $\frac{1}{21x^2-13x+2}+\frac{1}{28x^2-15x+2}=12x^2-7x+1$.

(2) $\left(\frac{x-a}{x+a}\right)^{\frac{1}{2}}+\left(\frac{x+a}{x-a}\right)^{\frac{1}{2}}=\sqrt{3}$.

(3) $x+y+z=5, \frac{x+ay}{y+bz}=\frac{y+az}{z+bx}=\frac{z+ax}{x+by}$.