A METHOD OF FIGURING REACTIONS FOR CONTINUOUS BEAMS.

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THE subject of continuous beams has received considerable attention in several of the latest text-books. The methods usually employed, however, are not, in all cases, conducive to their practical application. Only a limited number of cases for different loadings can be found already worked out and for this reason a rigid application of the theory is often avoided in practice. Rapidity, accuracy and ease with which results are obtained, are the requisites of any engineering practice, and it is hoped that the following method will promote all of these, as well as stimulate those who are interested to add more to what is here presented.

The general equation for the theorem of three moments for beams of constant modulus of elasticity and moment of inertia can be written as follows: (See article by Prof. Slocum in the Engineering News, Feb. 19, 1914.)

$$M_1l_1 + 2\,M_2(l_1 + l_2) + M_3l_2 = -\Sigma P_1\,l_1^2\,(k_1 - k_1^3) - \Sigma P_2\,l_2^2(2k_2 - 3k_2^2 + k_2^3) - 6EI\left(\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right);$$

where E is the modulus of elasticity and I the moment of inertia of the sectional area. The remaining notation may be obtained from Fig. 1.

If the supports are on the same level, and the load P_1 is unity and P_2 is zero, then the formula becomes:

$$M_1l_1 + 2M_2(l_1 + l_2) + M^3l_2 = l_1^2(k_1 - k_1^3).$$

If P_1 is zero and P_2 is unity, it is,

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = l_2^2(2k_2 - 3k_2^2 + k_2^3)$$

In applying these equations to any particular case an influence line may be plotted for the various values of k which lie between 0 and 1. Take the case of a continuous girder with two equal spans. Consider the load of unity in the first span only. Since $M_1 = 0$ and $M_3 = 0$

$$2M_2(l + l) = l_1^2(k_1 - k_1^3)$$

$$M_2 = \frac{l}{4}(k_1 - k_1^3).$$

$$\text{Then, } l\,R = \frac{l}{4}(k_1 - k_1^3) + l\,(1 - k_1).$$

$R_1 = 1 - 5/4k + \frac{1}{4}k^3$ upwards
$R_3 = \frac{1}{4}(k - k^3)$ downwards
$R_2 = \frac{1}{2}(3k - k^3)$ upwards.

The following table gives the values of R_1, R_2, and R_3 for values of k from 0 to 1 with the load of unity in the first span only:

k	R_1	R_2	R_3
.0	+1.0000	+ .0000	—.0000
.1	+ .8753	+ .1495	—.0247
.2	+ .7520	+ .2960	—.0480
.3	+ .6318	+ .4365	—.0683
.4	+ .5160	+ .5680	—.0840
.5	+ .4062	+ .6875	—.0937
.6	+ .3040	+ .7920	—.0960
.7	+ .2108	+ .8785	—.0893
.8	+ .1280	+ .9440	—.0720
.9	+ .0572	+ .9855	—.0427
1.0	+ .0000	+1.0000	—.0000

From this table the influence line shown in Fig. 2 may be plotted.

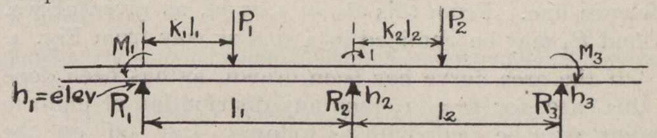

Fig. 1.

In this case it is not necessary to work out a table for the unit load in the second span because the curves are symmetrical, as may be seen in the figure. Since the influence line for any particular number of supports and ratio of spans is always the same, after it is once plotted it may be used to solve problems for any loading.

Problem 1.—Consider a continuous girder with two equal spans, with a uniformly distributed load of w pounds per linear foot over both spans. Since the product of the ordinate to the influence line at any point and the load at that point will give the value of the reaction for which the influence line is drawn, it is only necessary, in this case, to find the area under the curve. The areas below the horizontal axis are considered to be negative areas.

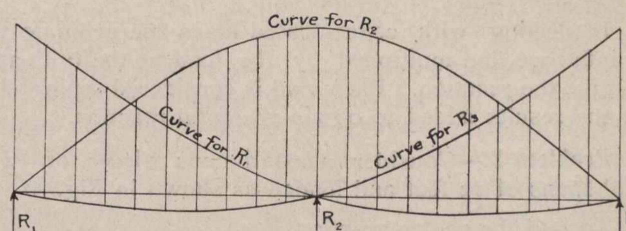

Fig. 2.—Influence Lines for Reactions, Two Equal Spans.

For those who are familiar with the calculus, it is sufficient to say that the area may be quickly obtained by integrating each curve from $k = 0$ to $k = 1$. This will be found to give the well-known values:

$R_1 = \frac{3}{8}wl$ upwards,
$R_2 = 5/4wl$ upwards,
$R_3 = \frac{3}{8}wl$ upwards.

The following graphical method is found to be more convenient as well as useful. It consists of drawing a curve such that the ordinate at any point will give the area under the influence line between the ordinate and the reaction R_1. This curve, which has been called the area curve for the lack of a better name, is also the same for any particular number of supports and ratio of spans and therefore when once obtained may be applied for any loading. The influence line is plotted first, as shown in Fig. 3, and each ordinate has been multiplied by w. The

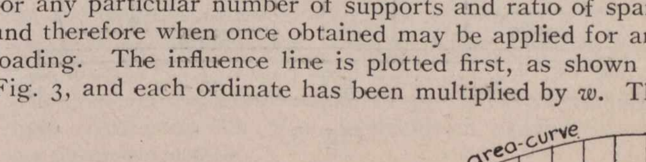

Fig. 3.—Area Curve for R_2.

middle ordinate is then drawn for each strip and horizontal lines are drawn to intersect the vertical ordinate at R_2. R_1 is chosen as a convenient pole and a new curve is constructed, as shown by drawing a string parallel to each ray in turn until it intersects the ordinate which bounds each strip on the left. The last ordinate on the new curve