

and the other produced in F , then AE , AC , AF are in harmonic progression.

Triangle ABC is isosceles and BC bisected in D . Draw BG parallel to AC , triangle DBG = triangle DCE in all respects.

$$\frac{FB}{BA} = \frac{FG}{GE} \text{ and } \frac{FE}{FG} = \frac{AE}{BG}$$

$$\therefore \frac{GE}{FG} = \frac{AE - CE}{(BG) = CE}$$

$$\therefore \frac{FB}{BA} = \frac{CE}{AE - CE}$$

$$\therefore \frac{AF}{AB} = \frac{AE}{AE - CE}$$

$$AF \cdot AE - AF \cdot CE = AB \cdot AE.$$

$$AF \cdot AE - AF(AC - AE) = AB \cdot AE.$$

$$AB = AC,$$

$$\therefore AB(AE + AF) = 2AF \cdot AE.$$

$$\therefore AC = \frac{2AF \cdot AE}{AE + AF}$$

$$\therefore AE, AC, AF \text{ are in H.P.}$$

13. Four points, moving each at a uniform speed, take 198, 495, 891, 1155 seconds respectively to describe the length of a given straight line. Supposing them to be together at any instant at the same end of the line, and to move in it from end to end continually, what interval of time will elapse before they are together at the same point again?

L. C. M. of numbers equal 62370,

$$\frac{62370}{198} = 315, \quad \frac{62370}{495} = 126,$$

$$\frac{62370}{891} = 70, \quad \frac{62370}{1155} = 54.$$

Each has made an even number of motions, except the first, in the time 62370 seconds, $\therefore$ time required will be

$$62370 \times 2 = 124740 \text{ seconds.}$$

PROBLEMS

for Entrance and Teachers' Examinations, by W. S. ELLIS, B.A., Mathematical Master, Cobourg Collegiate Institute.

1. A woman bought 12 yards of dress goods at 75 cents per yard, and 8 yards of lining at 20 cents per yard; but the clerk made a mistake and charged her with 8 yards

of dress goods and 12 yards of lining; who profited by the mistake, and how much?

Ans. The woman; \$2.20.

2. A dealer purchased a number of sheep and $1\frac{1}{4}$ times as many lambs; he paid the same sum of money for lambs that he did for sheep, and altogether he expended \$192. How much did he pay for each lamb, having given that he bought 54 sheep and lambs?

Ans. \$3 $\frac{1}{2}$.

3. The average attendance at a school during a half-year, consisting of 150 teaching days, was 88; there were 112 names on the roll; how many absences were recorded? What was the average number of absences for each pupil?

Ans. 3600; 32 $\frac{1}{2}$.

4. A certain quantity of zinc was melted with 20 pounds of lead, the zinc being $\frac{1}{4}$ of the united mass; what fraction of the weight of lead was that of the zinc.

Ans. $\frac{1}{4}$.

5. Find the smallest square number that will exactly contain each of the divisors, 5, 6, 7, 8, 9, 10.

Ans. 176400.

6. What is the smallest number to which if 3 be added, either 5, 6, 7, 8, 9, or 10 may be subtracted from the result an exact number of times?

Ans. 2517.

7. A man purchased a watch and chain, giving \$110 more for the former than the latter; and $\frac{1}{4}$ of the price of the watch was $2\frac{3}{4}$ times that of the chain: determine the value of each.

Ans. Watch, \$140; chain, \$30.

8. Two articles are together worth \$37, the difference of their prices is \$13; find the value of each.

Ans. \$12 and \$25.

9. It costs 20 cents per line to insert an advertisement in a newspaper the first time, 5 cents a line for each of the 10 subsequent insertions, then $2\frac{1}{4}$ cents a line always after that; now if a man is charged \$4.80 for a space of 6 lines, how often should his advertisement have appeared?

Ans. 15 times,

10. A merchant purchased a certain number of pounds of an article for \$12, had he purchased 10 pounds more he would have been allowed a discount of 10 per cent. on the