UNIVERSITY WORK.

MATHEMATICS.

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First Class Teachers—Grade C.

ALGEBRA.

1. If $x^n + ax^{n-1} + \ldots + sx + t = 0$, explain the principle upon which we proceed to find, if possible, a rational binomial divisor.

Find three such divisors in the equation, $x^6 - 4x^5 - 6x^4 + 18x^3 + 17x^2 + 22x + 24 = 0$.

1. Bookwork (Newton's Theory of Divisors).

Applying the test in this case, we find the divisors to be $x+2$, $x-3$, $x-4$.

2. Express $m^4 - 4m^3n + 5m^2n^2 - 2mn^3 + n^4$ as a rational integral function of p and n, where $p = m - n$.

2. Substituting and expanding, we have for result $p^4 - p^2n^2 + n^4$.

3. If y is a rational integral function of x, and y becomes zero when a is substituted for x, prove that $x - a$ is a factor of y.

Resolve into factors:

$$x^3 - \left\{a(a-b) + b(b-c) + c(c-a)\right\}x + \left\{ab(a-b) + bc(b-c) + ca(c-a)\right\}.$$

3. Bookwork.

$$(x - \overline{b-c})(x - \overline{c-a})(x - \overline{a-b})$$

4. If $\frac{a}{b} = \frac{c}{d}$ prove that $\frac{ma+nb}{ma-nb} = \frac{mc+nd}{mc-nd}$.

If $$\frac{x^2 + \left\{\frac{m+n}{m-n} + \frac{m-n}{m+n}\right\}xy + y^2}{x^2 + \left\{\frac{m+n}{m-n} - \frac{m-n}{m+n}\right\}xy - y^2} = \frac{v+1}{v-1}$$

find the simplest expression for the value of v.

4. Bookwork. By adding and subtracting 1 from each side of the equation and then dividing equals by equals, we get

$$\frac{m+n}{m+n} \cdot \frac{x}{y} = v.$$

5. What is a ratio? Does the ratio of two quantities depend upon their magnitude?

Given $y^3 + x^2y - a^2x^3 = 0$, to find the ratio of x to y when x becomes indefinitely great.

5. Bookwork.

6. What is meant by a *maximum* or a *minimum* solution?

It is required to divide a number a into two parts such that the quotient arising from dividing their product by the sum of their squares may be a *maximum*. Determine the quotient, and the division of the number required to produce it.

6. Suppose a to be the number, and let one part be x, then $\frac{(a-x)x}{(a-x)^2 + x^2} = p$: solving in the usual way, for maximum or minimum, $p = \frac{1}{2}$, or $-\frac{1}{2}$; take $p = \frac{1}{2}$, and we have $x = \frac{1}{2}a$ for maximum.

7. In an arithmetic series, find an expression giving the last term in terms of the first term the common difference and the sum of the series.

The nth terms of two *AP's* are respectively $\frac{1}{3}(n+2)$ and $\frac{1}{3}(3n-1)$. The same number of terms being taken in each series, what is the number when the sum of the second series is four times that of the first?

What is the *greatest* ratio of the sum of any number of terms of the second series to the sum of the same number of terms of the first?

7. Find the value of

$$n \text{ from } s = \frac{n}{2}\left\{2a + (n-1)d\right\},$$

and we have

$$l = \frac{2s - a}{n} = \frac{2s - a}{\frac{1}{2}d\left\{2a - d \pm \sqrt{(2a-d)^2 + 8ds}\right\}}$$