atmospheric pressure, would fill a hollow cube, one of whose edges is $\sqrt[3]{3.1416}$ feet. What is the pressure of the air in the cylinder on a square foot of the surface?

16. (*a*) One end of a uniform beam rests on the ground, the other end being supported in the hand of a man who exerts a pressure which is at right angles to the length of the beam, and in a vertical plane passing through the beam. Mention the different forces by which the beam is kept at rest.
* (*b*) If the weight of the beam be 200 lbs., and its inclination to the ground 60°, what force does the man exert.

NOTE.—It is recommended to the local examiners, that, in the above paper, the percentage of marks necessary, by the regulations of the Council of Public Instruction, in order that a candidate may be ranked of a certain grade, be taken on the full value of the paper, diminished by the number of marks allotted to the questions marked (*).

CHEMISTRY, BOTANY AND PHYSIOLOGY.—SECOND CLASS.

1. Distinguish an Elementary *Substance* from a Compound. Describe clearly what is meant by the *combining weights* of the Elements.
2. Give the symbols and combining weights of the fifteen elements which we have mainly to consider in Ag. Chemistry.
3. State and illustrate the law of *multiple proportions*.
4. What is Chemical Nomenclature? Give the force of the principal prefixes and terminations it employs.
5. State the principal properties of O. What different effects result from the union of O. with metalloids and metals?
6. Give the leading properties of chlorine. Account for its bleaching and disinfecting powers.
7. State what you know of the following: (1) Phosphuretted Hydrogen. (2) Caustic Potash. (3) Hydraulic Cement.
8. Name the organs of reproduction in plants, and describe their functions.
9. Give and fully describe the principal parts of the flower.
10. "In the adaptation of the food to the wants of the body, it is subjected to five different changes." Explain this statement fully.
11. Give the anatomy of the skin. Mention some of the uses of the cuticle.
12. Give briefly the hygiene of the nervous system.

OPTIONAL PAPER FOR SPECIAL CERTIFICATE IN

NATURAL HISTORY, BOTANY, AND AGRICULTURAL CHEMISTRY.

1. How are the Nematoneura distinguished from Acrita? Name their classes.
2. Name the classes of the Heterogangliata and give their leading characteristics.
3. Enumerate the *orders* of the Aves, the *tribes* of the Passeres, and state how the Scansores differ from the other tribes of this order.
4. Name the parts of the pistils and stamens of a flower, and give their uses.
5. What are Perennial plants? Describe their mode of life.
6. "There are two great classes of stems, which differ in the way the woody part is arranged in the cellular tissue." Fully explain this.
7. Describe the functions of the leaves. How are leaves classified as to their *veining*.
8. Name and describe the organic constituents of plants.
9. How can you distinguish the organic from the inorganic constituents of soils, and determine their relative proportions?
10. "All kinds of manure should be under cover,"—explain the reason of this: if exposed to the open air, how would you prevent the escape of valuable fertilizing elements?
11. How may the defects of sandy soils be remedied? Those of argillaceous soils?
12. State the beneficial effects of drainage on soils.

COMPLETE ALGEBRAIC PROOF OF THE BINOMIAL THEOREM.

BY J. C. GLASHAN.

For the Journal of Education.

(NOTE.—In the following paper = must be read "is (are) identically;" and R $(y,^2\ y^3 - - -)$ means "remaining terms in y^2, y^3, &c." R has no reference to a numerical value.

If n be a positive integer, by actual multiplication (or by induction if preferred),

$$(x^a - y^a)\left\{x^{a(n-1)} + x^{a(n-2)}y^a + x^{a(n-3)}y^{2a} + - - - - - - - - + x^a y^{a(n-2)} + y^{a(n-1)}\right\} = x^{na} - y^{na}. \quad (1)$$

$$\therefore (x+y)^n - x = \left\{(x+y) - x\right\}\left\{(x+y)^{n-1} + (x+y)^{n-2}x + (x+y)^{n-3}x^2 + - - - - - - - - + (x+y)x^{n-2} + x^{n-1}\right\}$$

$$= y\left\{(x+y)^{n-1} + (x+y)^{n-2}x + - - - - - + x^{n-1}.\right\}$$

$$\therefore (x+y)^n = x^n + y\left\{(x+y)^{n-1} + (x+y)^{n-2}x + - - - - + x^{n-1}.\right\}$$

Similarily develop each term in $(x+y)^{n-1} + (x+y)^{n-2}x +$ &c., and collect

$$\therefore (x+y)^n = x^n + nx^{n-1}y + R(y^2, y^3, - -) \quad (2)$$

By (1),

$$(x+y)^{-n} - x^{-n} = \left\{(x+y)^{-1} - x^{-1}\right\}\left\{(x+y)^{-n+1} + (x+y)^{-n+2}x^{-1} + - - - - + (x+y)^{-1}x^{-n+2} + x^{-n+1}\right\};$$

but $(x+y)^{-1} - x^{-1} = -yx^{-1}(x+y)^{-1}$.

$$\therefore (x+y)^{-n} = x^{-n} - y\left\{(x+y)^{-n}x^{-1} + (x+y)^{-n+1}x^{-2} + - - + (x+y)^{-2}x^{-n+1} + (x+y)^{-1}x^{-n}\right\}$$

Similarly develop each term in $(x+y)^{-n}x^{-1} + (x+y)^{-n+1}x^{-2}$ + &c., and collect,

$$\therefore (x+y)^{-n} = x^{-n} - nx^{-n-1}y + R(y^2, y^3, - - -) \quad (3)$$

By (1), m being positive or negative—

$$(x+y)^{\frac{m}{n}} - x^{\frac{m}{n}} = \left\{(x+y)^m - x^m\right\} \div \left\{(x+y)^{\frac{m(n-1)}{n}} + (x+y)^{\frac{m(n-2)}{n}}x^{\frac{m}{n}} + - - - - - + (x+y)^{\frac{m}{n}}x^{\frac{m(n-2)}{n}} + x^{\frac{m(n-1)}{n}}\right\}$$

$\therefore$ by (2) or (3)—

$$(x+y)^{\frac{m}{n}} = x^{\frac{m}{n}} + \left\{mx^{m-1}y + R(y^2, y^3, - - -)\right\} \div \left\{(x+y)^{\frac{m(n-1)}{n}} + - - - - + x^{\frac{m(n-1)}{n}}\right\}$$

Similarly develop each term in the denominator and collect

$$(x+y)^{\frac{m}{n}} = x^{\frac{m}{n}} + \frac{mx^{m-1}y + R(y^2, y^3, - - -)}{nx^{\frac{m(n-1)}{n}} + R(y, y^2, - - - - - -)}.$$

By actual division,

$$\therefore (x+y)^{\frac{m}{n}} = x^{\frac{m}{n}} + \frac{m}{n}x^{\frac{m}{n}-1}y + R(y^2, y^3, - - - - -) \quad (4)$$

$\therefore$ by (2), (3), (4), for any portensive (real) value of n

$$(x+y)^n = x^n + nx^{n-1}y + R(y^2, y^3, - - - - -) \quad (5)$$

Let $f(x) = Ax^a + Bx^b + Cx^c +$ &c.

$\therefore f(x+y) = A(x+y)^a + B(x+y)^b + C(x+y)^c +$ &c., (expanding by (5)) $= Ax^a + Bx^b + Cx^c +$ &c., $+ \left\{Aax^{a-1} + Bbx^{b-1} + Ccx^{c-1} + \text{\&c.}\right\}y + R(y^2, y^3, - - -)$ (6)

Let $f'(x)$ symbolize the coefficient of y in (6); and $f''(x)$, the function formed from $f'(x)$ as it form $f(x)$; $f'''(x)$, that similarly formed from $f''(x)$, &c.; thus: $f(x+y) = f(x) + f'(x)y + R(y^2, y^3, - - -)$

Multiply both sides into $z - y$ and arrange the right hand in terms of y—

$$f(x+y)(z-y) = f(x)z + \left\{f'(x)z - f(x)\right\}y + R(y^2, y^3, -) \quad (7)$$

$$(x+z)^n = x^n + \frac{(x+z)^n - x^n}{(x+z) - x}z$$

Since this is an identity, x and z are wholly independent of each other and may have any values given them. Substitute $(x+y)$ for x and $(z-y)$ for z; $(x+z)$ thus remaining unchanged.