UNIVERSITY WORK.

MATHEMATICS.

ARCHIBALD MACMURCHY, M.A., TORONTO, EDITOR.

CAMBRIDGE PROBLEMS.

NOVEMBER, 1882.

Prove the identity

$$(ax+by+cz)^3+(ay+bz+cx)^3+(az+bx+cy)^3 - 3(ax+by+cz)(ay+bz+cx)(az+bx+cy) = (a^3+b^3+c^3-3abc)(x^3+y^3+z^3-3xyz)$$

we have $(x^3+y^3+z^3-3xyz) = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega)^2$

where $1, \omega, \omega^2$ are the roots of $x^3-1=0$, and $\omega^3=1$.

Denoting $ax+by+cz$ by X, $bx+cy+az$ by Y, etc., we have

$$X^3+Y^3+Z^3-3XYZ=(X+Y+Z)(X+\omega Y+\omega^2 Z)(X+\omega^2 Y+\omega Z)$$

$$=(a+b+c\ x+y+z)(a+\omega b+\omega^2 c)(x+\omega y+\omega^2 z)(a+\omega^2 b+\omega c)\ .\ (x+\omega^2 y+\omega z)$$

$$=(a^3+b^3+c^3-3abc)(x^3+y^3+z^3-3xyz).$$

Q.E.D.

Effecting the reductions thus

$$\omega^2 a+\omega b+c=\omega^3(\omega^2 a+\omega b+c)$$
$$=\omega^2(a+\omega^2 b+\omega c)$$

etc. = etc., $\because \omega^3=1$.

ARITHMETICAL PROBLEMS.

L. B. Davidson, H. M. P. S., Glenallan.

1. Simplify $\dfrac{\frac{19}{2}}{7\times\frac{3}{1\frac{2}{3}}}\times\dfrac{7735}{67184}\div$

$$(1.1875-.97916\dot{6})+(\tfrac{1}{3}\text{ of }6\tfrac{1}{4}+\tfrac{1}{16}-2-\tfrac{3}{4}\text{ of }3\tfrac{1}{3})\ \div\ \frac{12\text{ lbs. }6\text{ oz., Avoir.}}{12\text{ lbs. }6\text{ oz., Troy.}}$$

Ans. 1.

2. Along a certain railway 96 miles in length there is a distance post every ½ mile, and a telegraph pole every 66 yds. How many of the telegraph posts might serve as distance posts *Ans.* 65.

3. A druggist buys 15 lbs. of opium by Avoir. weight at 50c. per oz. Find selling price per oz., Troy, that he may gain $\frac{5}{24}$ of his outlay. *Ans.* 75c.

4. If 9 men or 12 boys can do ⅜ of a work in 4½ hours, in what time will 10 men or 15 boys do the rest, the last ¼ of the work increasing 3½ times in difficulty?

Ans. 6 hrs.

5. A and B sold between them 50 horses. A sold his at $95.50 each; B sold his at $104.75 each. They received together $22.50 less than if they had sold them all at $101.50 each. Find the number of horses sold by each. *Ans.* 20, 30.

6. A bankrupt's assets are $1550, out of which he pays 75c. on the $ on all debts above $200; 62½c. on the $ on all debts from $100 to $200, and 50c. on the $ on all others. His creditors find upon examination that ⅓ his debts are above $200, and ⅛ between $100 and $200. Find his liabilities. *Ans.* $2400.

7. An employer who has been accustomed to hire men at $1.20 per day, finds he can save $\frac{1}{16}$ of his former outlay by engaging boys instead, each of whom can do only ⅔ of a man's work. Find the daily wages of the boys. *Ans.* 75c.

8. A dealer in liquors buys a hhd. of wine at $150, and after keeping ⅓ of it to sell by the "glass" at 5c., he bottles the rest off into an equal number of quart, pint and half-pint bottles. How many dozen of each has he, and what must be his selling price per dozen to gain on the whole $54.75? 1 wine glass $= \frac{5}{16}$ pts.

Ans. 8 doz.; $9, $4.50, $2.25.

9. An oarsman finds he can go as far down stream in 50 mins. as he can up stream in 70 mins. Compare the oarsman's rate with that of the stream. *Ans.* 6 : 1.