It is evident that if the position of the points $K$ and $J$ can be determined, the curve of pressure also becomes determinate,* and consequently the value of the thrust $T$. These are the ***Determining points*** in the curve, but unfortunately we have no means of deciding their position *a priori*. The reason for this is only too apparent. We have an external moment produced by the load; but no equivalent moment of resistance is developed in an arch ring as it would be in a metal rib. Hence until the points of application $K$ and $J$ can be determined, the equation between the moments of $T$ and $W$ remains indeterminate, and we cannot therefore proceed either to an analytical or a graphical solution. This abundantly accounts for the great variety of opinions amongst authors. All the theories of the arch can be classified in accordance with the positions they attribute to these points. In mathematical works it is often very difficult to discover the reasons for the choice made; and in works treating the subject graphically, while there is ample explanation of the method of drawing the curve of pressure, there is usually a lack of clearness as to the principles upon which the position of these determining points depends. Although an author may give good reasons for their position in one case, he may be too hasty in making a general application of the result to all cases. It is for this reason that some theories have fallen into discredit, although they undoubtedly have some good features in them, and may be quite applicable under certain conditions.

The direct determination of the position of these points has been attempted at about the same time by Scheffler and Dupuit, based upon the principle of least resistance, but applied in very different ways. This principle is due to Moseley (8) and may be briefly stated thus: the thrust developed at the key will not be greater than the amount which is just sufficient to maintain equilibrium. As it is evident that the least amount of thrust will correspond with the co-incidence of $K$ with $A$, and $J$ with $D$, Moseley takes the curve of pressure as tangent to the extrados and intrados at the key and joint of rupture respectively. Scheffler (9) generalizes the same view, and takes a curve of pressure tangent to the extrados and intrados of the arch ring for all cases of loading, whether symmetrical or not. Such a curve corresponds obviously to the case of a structure in which both the arch ring and abutments are everywhere on the point of giving way. If the abutments are about to yield, and the joints $CD$ and $AB$ are about to open, we have the required conditions, and failure will take place in the way so often represented in text-books. Although it can be shown that the curve will only take this position in extreme cases, it may nevertheless be useful as a means of investigation. This theory is also in favor with experimentalists, (10) as it accords with the final conditions which can be observed at the moment of failure. An arch might be designed in accordance with it, by giving it minimum dimensions throughout, and afterwards adding material to cover the possibly greater values of the thrust and other pressures that might occur.

Dupuit (11) applies the same principle but proceeds more carefully. He brings it into relation with the ordinary process employed in the building of arches, and the effects that can be observed when the centres are struck. By following the steps he takes, we will be best able to judge of the principle, and to determine the range of its applicability to the arch.

The following summary of his reasoning is taken from Claudel: (12) While the arch remains on its centres the pressure in the arch ring is zero throughout; and as the centres are eased, the pressures found in the finished arch are developed. These pressures therefore must pass through all the intermediate values from zero upward; and in accordance with the principle of least resistance, he infers that when once the least amounts necessary for stability are reached, there is no further reason why the pressures should continue to increase. In the movement which occurs when the centres are struck, there is something fixed and certain which depends only upon the curve of the arch, and also an amount of uncertainty depending upon the nature of the materials and the methods of construction, so that the uncertainty finally remaining is reduced to such narrow limits as to have no further interest from a practical point of view. While the arch remains on its centres the curve of equilibrium is entirely beneath the arch, and is determined by the intersections of the verticals through the successive centres of gravity, with the production of the joint lines. (Fig. 4.) This may be called the curve of static equilibrium, and corresponds with an absence of pressure in the arch ring. This curve serves merely to indicate the point in the arch around which rotation would tend to take place. As a small thrust develops at the key, this curve rises toward the arch and indicates more distinctly the position of the point of rotation $D$. When the thrust increases sufficiently to make the curve of pressure pass through the point $D$, it is evident that it is sufficient to maintain equilibrium. There is therefore no reason why the curve should pass this

*As the curve of pressure is symmetrical for symmetrical loading, and the points $K$ and $J$ imply direction as well as position, they are mathematically equivalent in the complete arch to six determining points on the curve. It would therefore have to be of a very high order to take two different positions between $K$ and $J$.