cessful, makes it the property of all; and we now arrive at the third stage of the process, which is the simultaneous answer properly so called. The teacher reverts to the points which was the subject of the first indiscriminate answer; he presents it again, either as a direct question, or as an ellipsis, and the answer ought now to be given by all, or nearly all the children. The simultaneousness of this is the test of success in the preceding part of the process; and if it is not attained, the illustration by analogies, &c., must be recommenced, however great the demand may be on the teacher's patience; and the success of this further illustration must be tested again in the same way. Every successive step in the lesson ought to form the subject of a simultaneous answer, and be thus fixed in the minds of the children. No collective lesson can be given with life and effect where this is overlooked. It will infallibly degenerate into a lecture, or into something not so efficient as that.

When the proper simultaneous answer has been obtained, we shall find employment for individual answering. It is in comparatively rare cases that every child can be got to answer with all the rest, by any amount of teaching skill. There are always to be found a number of children, who from constitutional dullness, or habitual idleness, cannot be got to keep pace with their school-fellows; and of course every teacher knows where to look for them in his own school. After every simultaneous answer, one or more of these should have his attention and exertions quickened by a question addressed to himself; and the teacher should ever be on the alert to see where they are most needed. If in addition to these he should now and then select others less markedly deficient, or even now and then the more advanced scholars, the results will be beneficial. He will be saved from making too high an estimate of the progress of favourite scholars; and will now and then discover where he needs to be more exact or more lucid, even for his most advanced pupils.

In illustration we take the liberty of subjoining a specimen of what we desire from the lesson on the Mole, in Mr. Stow's training system. The italics mark the words supplied by the children, and the dots indicate the places where ellipses are made.

"When you look at a land bird and a water bird, and compare them, what do you notice? *A great difference in the way in which they are made.* What was the word formerly given instead of 'the way in which they are made?' Try to remember. *Structure.* Quite right; and they are made differently, or have a different...*structure*...because they differ in their...*ways of living*...or their... Who remembers the word that means ways of living? *Habits.* Now all sit upright and attend. When you find an animal of a particular structure, what will you be led to think about it? *That it has particular habits.* And if you are told that an animal lives in an uncommon place, or has particular habits such as the mole, what will you expect it to be? *Of a particular structure.* All will now answer me. The form or structure of the animal is always well...*fitted to its way of living*... All again. The habits and structure of the animal always...*agree — suit one another very well*... We will now hear this boy in the lowest seat repeat it........ Quite correct."

We consider this as a fair specimen of the way in which a skilful teacher makes good each successive advance in a collective lesson; but to exemplify the varied modes in which the second part of the process, that of simplification and illustration, may be conducted, we must refer to the lesson as it stands in Mr. Stow's work.

Since it is in collective lessons, both sacred and secular, that the mind of the principal teacher can be brought most directly in contact with the understanding and moral feeling of the children, no pains should be spared to perfect our modes of communication in such lessons; and we know of no instrument of collective teaching which rivals this in power. If in the large circle of professional men by whom this publication is read, these few remarks render this mode of teaching better understood; or more easily practised, we shall feel amply repaid for thus putting on record the results of many years' experience and careful observation.—*Papers for the Schoolmaster.*

THE PROPERTIES OF NUMBERS.

Curious properties of numbers do not lie immediately in the track of the mathematical investigator. There will probably, however, be a new light thrown on them at no distant period; and those strange and beautiful laws whose uses we cannot ascertain will be justified even to utilitarians.

Dr. Brooth's law, in reference to numbers of six places, may be much extended; and there is most likely some general theorem, yet to be discovered, on which all its extensions depend. The following, which (simple as they are) I do not remember to have seen any where enunciated, may be productive of some interest to the younger mathematical readers of the *Journal of Education.*

I. Any number of eight places consisting of any four figures repeated, is divisible by 73 and 137. *Ex. gr.:*—

$$98329832 \div 73 = 1346984;$$
$$11021102 \div 137 = 80446.$$

II. Any number of eighteen places, consisting of any nine figures repeated, is divisible by 7, 11, 13, and 19.

III. Any number of sixteen places, consisting of any eight figures repeated, is divisible by 17, &c.*

The "Self-proving Examples" just published by Mr. Alexander Ellis, I have not seen; such a work, well executed, cannot be otherwise than valuable. The theory of fractions—most important of arithmetical subjects—requires copious practical illustration: and there are one or two fractional formulæ which I have found useful for furnishing multitudinous examples. *Ex. gr.:*—

$$\frac{2}{3}+\frac{5}{6}+\frac{8}{10}+\ldots\ldots=\frac{n(n+1)}{n+2}$$

Thus $$\frac{2}{3}+\frac{5}{6}+\frac{8}{10}+\frac{14}{15}+\frac{20}{21}=\frac{5\times 6}{7}=4\frac{2}{7}$$

$$4\frac{2}{7}+\frac{27}{28}+\frac{35}{36}+\frac{44}{45}=\frac{8\times 9}{10}=7\frac{1}{5}$$

Again, $$\frac{2}{15}-\frac{3}{35}+\frac{4}{63}-\ldots\ldots=\frac{n}{12n+9}$$

when an even number of terms is taken and n = half that number. This formula is very serviceable with the aid of Barlow's Tables of Squares, &c., thus:—

$$\frac{13}{165}+\frac{28}{3135}-\frac{29}{3363}=\frac{14}{177}$$

$$\frac{100}{1209}+\frac{202}{163215}-\frac{203}{164835}=\frac{101}{1221}$$

Combinations of the various fractional processes are easily managed by algebraic formulæ; and, as De Morgan observes, if the boy detect the secret, he is fit for something more difficult. Indeed, it is a good algebraic exercise to give the detection of the formula as a problem to be solved. The following is an easy instance, by no means unserviceable:—

$$\frac{\frac{a}{x}}{a-\frac{a}{x}}=\frac{1}{a-1}$$

Ex. gr.:— $$\frac{9\frac{2}{17}}{11\frac{5}{23}}\div\left(9\frac{2}{17}-\frac{9\frac{2}{17}}{11\frac{5}{23}}\right)=\frac{23}{235}$$

—MORTIMER COLLINS *in English Journal of Education.*

Miscellaneous.

WRITTEN EXAMINATIONS.

Frequent written reviews are among the most successful means that teachers can employ for securing thoroughness and accuracy of scholarship. Several topics are written distinctly on the blackboard, and the pupils are required to expand them as fully and accurately as possible. Each pupil is seated by himself, and furnished with pen and paper; but receives no assistance, direct or indirect, from either teacher or text-book. This mode of examining a class accomplishes at least three important objects at the same time. It affords a thorough test of the pupil's knowledge of the subject; it is one of the best methods of cultivating freedom and accuracy in the use of language; and it furnishes a valuable discipline to the pupil's mind, by throwing him entirely on his own resources. The task of examining so many separate written exercises, and of estimating their value, increases the labor of the teacher, but the gain to the pupil is more than an equivalent for the extra service required.—*Mass. Teacher.*

VENTILATION.

In the process of respiration, a full grown man draws into his chest about 20 cubic inches of air; only one-fifth of this is oxygen, and nearly one-half of this oxygen is converted into carbonic acid. Now, allowing fifteen inspirations per minute for a man, he will vitiate about three hundred cubic inches, or nearly one-sixth of a cubic foot of atmospheric air, and this, by mingling as it escapes with several times as much, renders at least two cubic feet of air unfit for respiration. Now the removal of this impure air, and the bringing in of a constant fresh supply, have been provided for by nature in the most perfect manner, and it is by our ill-contrived, artificial arrangements

* The following properties—for which *vide* Wood's Algebra—are curious:—
Any number divided by 6 leaves the same remainder as its cube divided by 6.
The difference of the squares of any two odd numbers is divisible by 8.
The difference of the squares of any two prime numbers, above 5, is divisible by 24.
Any number of 4 digits is divisible by 7, if the first and last digits be the same, and the digit in the place of hundreds double that in the place of tens.