sashes, with five sections of clear glass in zinc channels so standing passengers can see out. There are three windows across the ends of the car, affording passengers an unobstructed view in either direction.

The motor cars are equipped with four Westinghouse, No. 93A, 60-h.p., 600-volt, d.c. motors, straight air brakes and Baldwin trucks. They weigh complete about 28 tons.

ASTRONOMICAL OBSERVATIONS FOR AZIMUTH ON DOMINION LAND SURVEYS.

By J. A. Macdonald.*

With the high class of instruments now supplied our Dominion land surveyors, Azimuth observations, by using Polaris, are possible at almost any time on a light, clear, day, provided the telescope is properly focused. During the summer difficulties are sometimes experienced in detecting the star, when the object glass is not sufficiently protected from the reflection of the sun's rays. The instrument makers, however, provide a ray shade to obviate this.

On every clear day the watch error is easily determined by the surveyor by the sun's transit at noon. This observation entails no delay, and is by far the safest, because an observation on a star for the same purpose might keep the surveyor late on the line, but often prove impossible later on in the day.

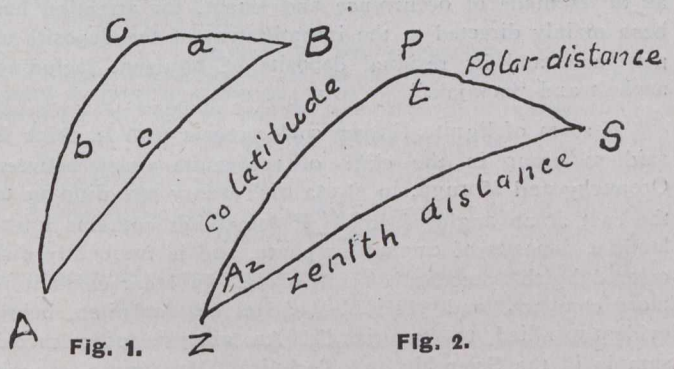

Fig. 1.

Fig. 2.

The formula recommended by the Surveyor-General's Department for the reduction of Azimuth observations in Polaris is derived as follows:

In the spherical triangle to be solved, two sides and the included angle are given. The sides are arcs equal to the co-latitude of the place of observation, and to the Polar distance of the star (Polaris).

They include the hour angle of this star at the moment of observation.

The required angle and Azimuth of Polaris is formed by the two great circles which intersect at the zenith, and of which one is the Meridian of the place.

In Fig. 1.—To prove $\tan A = \dfrac{\tan a \operatorname{cosec} b \sin c}{1 - \tan a \cot b \cos C}$

Now, $\dfrac{\sin A}{\sin a} = \dfrac{\sin C}{\sin c}$ or $\sin A \sin c = \sin C \sin a$ (1)

and $\cos a = \cos b \cos c + \sin b \sin c \cos A$,

or, $\sin c \cos A = \dfrac{\cos a - \cos b \cos c}{\sin b}$ (2)

Dividing (1) by (2) we get

$$\tan A = \frac{\sin a \sin b \sin C}{\cos a - \cos b \cos c}$$

Dividing numerator and denominator by $\cos a \sin^2 b$ we get

$$\tan A = \frac{\tan a \operatorname{cosec} b \sin C}{\dfrac{1}{\sin^2 b} - \dfrac{\cos b \cos c}{\cos a \sin^2 b}}$$

Now, $\cos c = \cos a \cos b + \sin a \sin b \cos C$

Hence,

$$\tan A = \frac{\tan a \operatorname{cosec} b \sin C}{\dfrac{1}{\sin^2 b} - \dfrac{\cos b}{\cos a \sin^2 b}\left(\cos a \cos b + \sin a \sin b\right)}$$

$$= \frac{\tan a \operatorname{cosec} b \sin C}{\dfrac{1}{\sin^2 b} - \dfrac{\cos a \cos^2 b}{\cos a \sin^2 b} - \dfrac{\sin a \sin b \cos b \cos C}{\cos a \sin^2 b}} =$$

$$= \frac{\tan a \operatorname{cosec} b \sin C}{\dfrac{1}{\sin^2 b} - \dfrac{\cos a \cos^2 b}{\cos a \sin^2 b} - \dfrac{\sin a \sin b \cos b \cos C}{\cos a \sin^2 b}}$$

$$= \frac{\tan a \operatorname{cosec} b \sin C}{\dfrac{1 - \cos^2 b}{\sin^2 b} - \tan a \cot b \cos C}$$

$$= \frac{\tan a \operatorname{cosec} b \sin C}{1 - \tan a \cot b \cos C}, \text{ since } 1 - \cos^2 b = \sin^2 b.$$

Now in the spherical triangle P Z S (Fig. 2).

P = Pole

Z = Zenith

S = Star (Polaris)

$$\tan P Z S = \frac{\tan P S \operatorname{cosec} P Z \sin Z P S}{1 - \tan P S \cot P Z \cos Z P S}$$

$$\text{or tan Azimuth} = \frac{\tan P \sec L \sin t}{1 - \tan P \tan L \cos t}$$

where P, L and t are Polar distance, latitude and hour angle respectively.

This is the formula recommended by the manual of surveys of Dominion lands for the reduction of Azimuth observations by using Polaris, though its derivation is not shown in the manual, but it is here given in graphic form.

Surveyors are, however, at liberty to use any formula or process for reducing their observations, but as forms and tables could not be prepared for every method this formula,

$$\tan Z = \frac{\tan P. \sec L. \sin t}{1 - \tan P. \tan L. \cos t}$$

where Z. P. L. t. are Azimuth, Polar distance, latitude and hour angle respectively.

The logarithms of secant and tangent L are given in tables for the north side of every section

By means of these astronomical field tables now supplied surveyors, the observation has become very easy and simple.

The instruments necessary are a watch, keeping sidereal time, that is to say, gaining 3 minutes 56 seconds a day, and a four-inch transit theodolite, now listed by several manufacturers, so-called "Canadian Pattern." A common watch of a reputable manufacturer is good enough. The best grades keep the time for weeks and months with sufficient

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