C in 5. A does half a certain piece of work in 12 hours; in what time can it be finished by B and C, working separately equal times, and C succeeding B?

A in 28 = B in 42 = C in 60, ∴ amounts are as 14:21:30. A does work in 24 hrs. ∴ B in 16 hrs., and C in $\frac{160}{7}$ hrs. In 1 hr. B does $\frac{1}{16}$ and C does $\frac{7}{160}$ work. In 1 hr. both will do $\frac{1}{16}+\frac{7}{160}=\frac{17}{160}$ work. If both worked together they would finish in $\frac{160}{17}$ hrs., but as they come one after the other they will require twice as long, $=\frac{320}{17}=18\frac{14}{17}$ hrs.

4. A note for \$500, made March 9, at 3 months, is discounted April 11, at 8%. What is received for the note? (True discount.)

59 days at 8% gives $\frac{59}{365}$ of $8=\frac{472}{365}$ of a dollar interest on each dollar of face value.

∴ Discount $=\frac{472}{837}$ and P. W. $=\frac{365}{837}$ of face value.

Then $\frac{365}{837}$ of \$500 = &c.

5. The unclaimed dividends on a certain amount of stock which pays 6% per annum amounted in 3 years to \$1152. The stock was sold at a discount of $12\frac{1}{2}$% on its par value. What sum was realized?

As no rate of interest is mentioned, we assume that the dividends do not accumulate.

3 years' dividends = \$1152 = 18% of stock = $\frac{18}{100}$ stock

∴ stock = 115200 ÷ 18. At a discount of $\frac{1}{8}$.

Value = $\frac{7}{8}$ of 115200 ÷ 18 = \$5600.

6. Teas at 3s. 6d., 4s., and 6s. a pound, are mixed to produce a tea worth 5s. a pound. What is the least integral number of pounds that the mixture can contain?

Marking the gains per pound with +, and the losses with −, we have

$$+\ 1\tfrac{1}{2},\quad +\ 1 \text{ and } -\ 1.$$

Now the total gain must equal the total loss. It is evident that 2lbs, 1lb, and 4lbs are the smallest *integral* numbers which will make the loss balance the gains. ∴ Ans. = 7lbs.

7. A man buys 150 pounds of sugar, and, after selling 100 pounds, finds that he has been parting with it a loss of 5%. At what rate per cent. advance on the cost must he sell the remaining 50lbs that he may gain 10% on the entire transaction?

To make 10% he needs to get the cost price of 165lbs.

He sold 100lbs for the cost of 95lbs.

He must sell 50lbs for the cost of 75lbs.

i.e., at the rate of 100 for 150, ∴ advance = 50%.

8. Each member of a pedestrian club walks as many miles as there are members in the club. The total expense is £50 13s. 11d. How many members are there?

If there are x members, each walks x miles, at x pence per mile, ∴ x^3 is the cost in pence of the whole trip = 12167 pence.

i.e., $x^3=23^3$, $x=23$ members.

9. The hour, minute, and second hands of a watch are on concentric axes. When first after 12 o'clock will the direction of the second hand produced backwards bisect the distance between the other two?

The rate of the extremities of the hand are as 1:12:720 respectively, for the hour hand goes round the circle once, the minute hand twelve times, and the second hand seven hundred and twenty times in the course of twelve hours. Suppose the hands in the position described in the question. The reader may draw a figure, placing A at the end of the hour hand, B at the end of the minute hand, and C at the extremity of the second hand. Let the second hand be produce backward half way between A and B. Mark the point D. Call the distance from XII. to D one space, then A to B is 11, and B to C 108 spaces. Also observe that B to C is the same as C to A.

∴ 11 spaces + 708 spaces + 708 spaces = circle = 60 min.

∴ 1 space = 60 min ÷ 1427 = $30\frac{390}{1427}$ sec.

ELEMENTARY ALGEBRA.

1. Find the factors of $(a-b)^5+(b-c)^5+(c-a)^5$.

Suppose a were to become $=b$, we should have left $(b-c)^5+(c-b)^5$ which is $=0$. This shows that when $a-b=0$ the whole expression $=0$, and from this we infer that the expression itself is of the form $(a-b)Q$ where Q is the exact quotient when $a-b$ is divided into the expression. For if $a-b$ did not divide the expression exactly but left a remainder, R, which has no longer contained $a-b$, then it would be of the form $(a-b)Q+R$, and when $a-b=0$ the first term would vanish but the remainder, R, would not vanish. Hence it is plain that if the whole expression becomes zero when any particular letters become zero, these letters must be exact factors. Thus, in the case before us $(a-b)$ is a factor, similarly $(b-c)$ and $(c-a)$ are factors, therefore their product $(a-b)(b-c)(c-a)$ is a factor of the given expression.

Now the given expression is of five dimensions, and therefore must have factors to make up five dimensions just as a^5 must $=a, a, a, a, a$. We have found three such factors, corresponding to a, a, a. We must have left two factors corresponding to a, a, or one factor corresponding to a^2. There is no other supposition possible. But we cannot have two other factors of one dimension like a and b, or like $a+b$, and $b-c$; for a, b, and c occur throughout the expression in precisely the same manner, *i.e.*, wherever there is an a^5 there is a b^5 and a c^5, etc. So that if a and b were factors c must also, from the symmetry of the expression, be a factor, and thus abc would be a factor, which with the three factors already found would give *six* dimensions instead of five. Similarly it is of no use to try $a+b=0$ as a factor. Therefore the remaining part of the expression must be a single factor of two dimensions.

Now a factor of two dimensions in a, b, and c can only contain terms made by taking a into a, b, and c, b into a, b, and c, and c into a, b, and c, *i.e.*, it can only have terms of the form a^2, ab, ac, b^2, bc, c^2. Looking at these we see that they are all of two kinds, viz.: *Squares* like a^2, b^2, c^2, and products like ab, bc, ca.

Hence the factor we are searching for must be of the form $K(a^2+b^2+c^2)\pm P(ab+bc+ca)$, where K and P are numerical and include any numerical factor that might belong to the rest of the expression. ∴ $(a-b)^5+(b-c)^5+(c-a)^5$ must

$$=(a-b)(b-c)(c-a)\{K(a^2+b^2+c^2)+P(ab+bc+ca)\}.$$

To find K and P, put $c=0$ and we have

$$(a-b)^5+(a^5-b^5)=(a-b)(-ab)\{K(a^2+b^2)+P(ab)\}.$$

Divide this by $a-b$, and

$$(a-b)^4-(a^4+a^3b+a^2b^2+ab^3+b^4)=-ab\{K(a^2+b^2)+P(ab)\}.$$

i.e., $-5ab(a^2-ab+b^2)=-ab\{K(a^2+b^2)+P(ab)\}$

or, $5(a^2+b^2)-5ab=K(a^2+b^2)+P(ab)$.

And as these are not only equivalent but *identically the same*

∴ K must $=5$ and $P=-5$.

So that the whole expression must

$$=(a-b)(b-c)(c-a)\{5(a^2+b^2+c^2)-5(ab+bc+ca)\}$$

i.e., expression $=5(a-b)(b-c)(c-a)(a^2+b^2+c^2-ab-bc-ca)$.

NOTE.—We have written down every step of this process because no explanation of this method is given in any of the ordinary text-books, and students generally experience considerable difficulty in obtaining a grasp of it. See *McLellan's Teachers' Handbook*, pp. 87 and 229, for a concise statement of it.

2. Factor $a^3(b^2-c^2)+b^3(c^2-a^2)+c^3(a^2-b^2)$.

As in (1) we see by inspection that $(a-b)(b-c)(c-a)$ is one factor.

Also as in (1) we see that the remaining part must be a single factor of two dimensions of the form

$$P(a^2+b^2+c^2)+Q(ab+bc+ca).$$

∴ $a^3(b^2-c^2)+b^3(c^2-a^2)+c^3(a^2-b^2)$

$$=(a-b)(b-c)(c-a)\{P(a^2+b^2+c^2)+Q(ab+\&c.)\}$$

Put $c=0$ and we get

$a^3b^2-a^2b^3=(a-b)(-ab)\{P(a^2+b^2)+Q(ab)\}$

or $a^2b^2(a-b)=$ " " " "

∴ $-ab=P(a^2+b^2)+Q(ab)$, which shows that $P=0$, and $Q=-1$

∴ Expression $=(a-b)(b-c)(c-a)(-ab-bc-ca)$.

We subjoin a few more examples for practice and additional ones may be found in the *Teachers' Handbook passim.*

3. $a^3(b+c-a)^2+b^3(c+a-b)^2+c^3(a+b-c)^2+abc(a^2+b^2+c^2)$
$+(a^3+b^3+c^3-bc-ca-ab)(b+c-a)(c+a-b)(a+b-c)$
$=2abc(bc+ca+ab)$.

4. $2(a+b+c)^3-(a+b)^3-(b-c)^3-(c+a)^3+3abc$
$=3(a+b+c)(ab+bc+ac)$.

5. $(y-z)^5+(z-x)^5+(x-y)^5$
$=5(x-y)(y-z)(z-x)(x^2+y^2+z^2-xy-yz-zx)$.

6. $(a-b)^4+(b-c)^4+(c-a)^4=2(a^2+b^2+c^2-ab-bc-ca)^2$.

7. $(a+b+c)^5-(a^5+b^5+c^5)$
$=5(a+b)(b+c)(c+a)(a^2+b^2+c^2+ab+bc+ca)$.

8. $a^4(b-c)+b^4(c-a)+c^4(a-b)$
$=-(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$.

O. O.