ARTS DEPARTMENT.

ARCHIBALD MACMURCHY, M.A., MATHEMATICAL EDITOR, *C. E. M.*

Our correspondents will please bear in mind, that the arranging of the matter for the printer is greatly facilitated when they kindly write out their contributions, intended for insertion, on one side of the paper ONLY, or so that each distinct answer or subject may admit of an easy separation from other matter without the necessity of having it re-written.

Mr J. B. McColl, Teacher, sent correct solutions of problems 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, and 155, last month, too late to be noticed in the November number.

SOLUTIONS

By the proposer, J. H. BALDERSON, B.A., Math. Master, High School, Mount Forest.

165. Prove that the equations

(1) $x+y+z=a+b+c$;

(2) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$;

(3) $\frac{x}{a^3}+\frac{y}{b^3}+\frac{z}{c^3}=0$;

are equivalent to only two independent equations if $bc+ca+ab=0$.

(1) $\frac{x}{a+b+c}+\frac{y}{a+b+c}+\frac{z}{a+b+c}=1$;

(2) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$;

(2) $-$ (1) then

$$\frac{x(b+c)}{a(a+b+c)}+\frac{y(a+c)}{b(a+b+c)}+\frac{z(a+b)}{c(a+b+c)}=0,$$

or $\frac{x(b+c)}{a}+\frac{y(a+c)}{b}+\frac{z(a+b)}{c}=0,$

or $\dfrac{\frac{x}{\frac{1}{a}}}{b+c}+\dfrac{\frac{y}{\frac{1}{b}}}{a+c}+\dfrac{\frac{z}{\frac{1}{c}}}{a+b}=0$;

by the restriction $a=\frac{-bc}{b+c}$,

or $\frac{1}{b+c}=-\frac{a^2}{abc}$

$\therefore \frac{1}{c+a}=-\frac{b^2}{abc}$ by symmetry,

$\therefore \frac{1}{a+b}=-\frac{c^2}{abc}$;

we have $\dfrac{\frac{x}{a^3}}{-abc}+\dfrac{\frac{y}{b^3}}{-abc}+\dfrac{\frac{z}{c^3}}{-abc}=0,$

or $\frac{x}{a^3}+\frac{y}{b^3}+\frac{z}{c^3}=0$;

$\therefore$ the three equations are equivalent to two independent equations.

167. The number of combinations of $2n$ things taken n at a time of which n and no more are alike is 2^n, and the number of combinations of $3n$ things of which n and no more are alike, is $2^{2n-1}+\frac{\lfloor 2n}{2(\lfloor n)^2}$.

n things can be taken from the n like things in one way; $(n-1)$ things can be taken from the n like things and 1 from the n unlike things in n ways; 2 things can be taken from the n unlike things and $(n-2)$ from the like in $\frac{n(n-1)}{\lfloor 2}$ ways, &c., &c.

It will be found that the whole number of ways is $1+n+\frac{n(n-1)}{\lfloor 2}+\frac{n.(n-1)(n-2)}{\lfloor 3}$ $+\ldots+1=(1+1)^n=2^n$.

Take n things from the like, this can be done in 1 way. Take $n-1$ things from the like and 1 from the unlike, done in $2n$ ways. Take $n-2$ things from the like and 2 from the unlike, done in $\frac{2n(2n-1)}{\lfloor 2}$ ways.

Take $n-3$ things from the like, and 3 from the unlike, done in $\frac{2n(2n-1)(2n-2)}{\lfloor 3}$ ways.