demonstration. Some engineers put 100 feet or 200 feet of a curve of larger radius at each end of the main curve, and trust to the trackman for the rest, others introduce a series of short arcs of decreasing radii, say 30 feet of 1° curve, 30 feet of 2° curve, etc., leading up to the main curve at the rate of 30 feet per degree; this necessitates placing the transit every 30 feet, is a tedious and clumsy method, and the result is that the trackmen fuse one portion into another until it is, to all intents and purposes, the same as a spiral. It does not admit of ordinary calculation or manipulation unless modified as in the next paragraph.

(*b*) In "The Railway Spiral," by Searles, is given a complete analysis of the transit work necessary to lay down a succesion of short circular arcs, beginning at zero, and having equal lengths of arcs of equal increments of sharpness, *e.g.*, 20 feet of 1° curve, 20 feet of 2° curve, etc., up to any required sharpness. Tables of deflections are worked out, so that any point of change of curvature can be used as a transit site, and any point of change can be established from any other point of change by transit deflections. Methods of conversion are also given, so that from one foundation series other deflection tables may be determined suitable for spirals of more or less rapid sharpening. The subject is well discussed and thoroughly worked out for all probable conditions, but as it does not present that same flexibility and simplicity of use which the cubic parabola possesses, its continued use is doubtful. It has served its day, and, where used, furnished the trackmen with a succession of hubs, really the ends of arcs of increasing sharpness, but practically points on a spiral very suitable for an easement curve.

(*c*) The Holbrook spiral (quadratic parabola). The idea involved in this easement curve is that the *vertical* acceleration of the train, as it passes around it, should be uniform. If we let t represent horizontal distances (with train moving at a uniform speed) in the general formula $s = \frac{1}{2}\,ft.^2$, then, in order to keep f (acceleration) constant, the distance, s, (*i.e.*) the amount which the train rises above the normal tangent level, must vary as the (distance)^2, and as the elevation should always bear a constant ratio to the degree of curve at each point, therefore the degree of curve on this required spiral must vary as the square of the distance from the zero of such a curve, (*i.e.*) the radius of curvature, at each instant, must vary *inversely* as the distance from the zero of the curve

A curve of such a nature has the equation $y = (f)x^4$ to represent it, and is a curve very flat at the beginning, but increasing very rapidly in curvature. This easement curve sacrifices the correct horizontal alignment, as will be seen in the next paragraph, for a supposed refinement in the vertical one; it is quite difficult to apply except in most ordinary cases, as the formulæ used involve expansions of sine and cosine, does not prevent any advantage over the cubic parabola, and is not so adaptable or easy to manipulate in the case of any problems having special conditions (*d*).

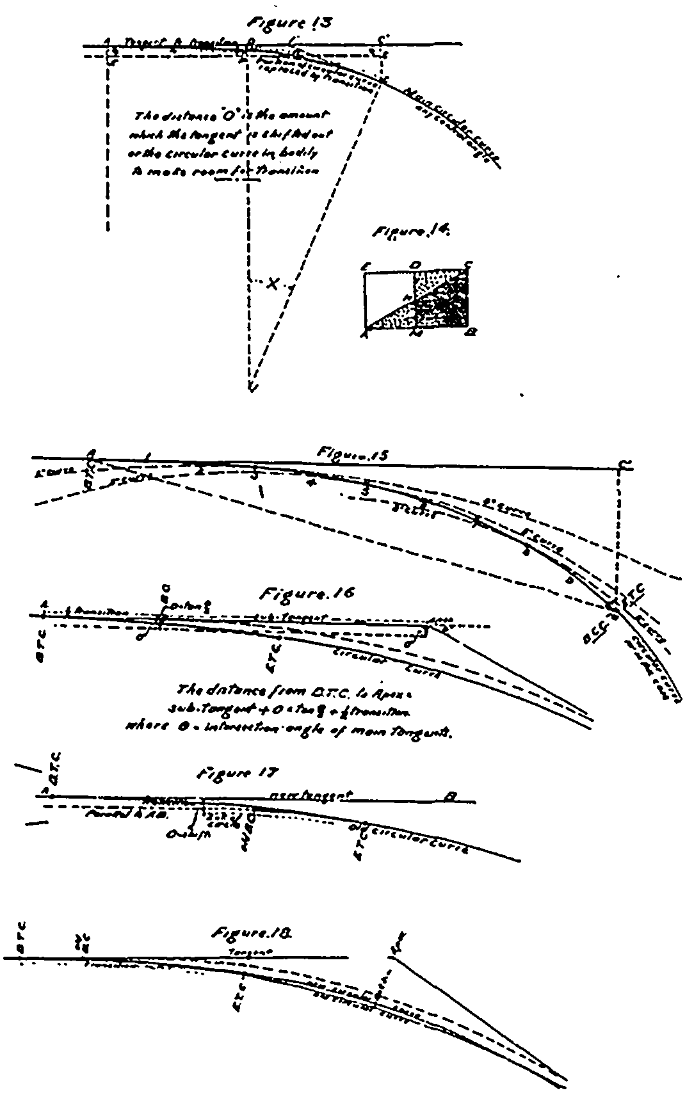
The Cubic Parabola.

This curve as adapted to transitions to railway circular curves has been studied pretty thoroughly. Howard, Armstrong, and others have written pamphlets on it; the transactions C.S.C.E. for 1891, 1892 and 1893 have several papers and discussions on it, and its probable originator, the late A. M. Wellington, determined very simple equations for it which were published in the *Engineering News*, January and February, 1890.

It is this last demonstration that will be now given to which will be added necessary developments. The curve required for a suitable transition is one which starting with an infinite radius or D (degree of curve) $= 0$, at the $B\ T\ C$ (A Figs. 13 and 14) has a degree of curve at each point in direct proportion to its distance from the $B\ T\ C$ until it joins and becomes tangent to the main curve at C, and is, at that point, of the same degree of curvature as the main curve.

The cubic parabola $y = (f)\,x^3$ approximates to these conditions.

Let $A\ M\ C$ (Fig. 13) be the cubic parabola, $A\ C^1$ tangent to it at A, and $I\ C$ the radius of the D degree curve with which it connects at C, having there a common tangent $I^1\ C$.



Let X be the central angle of the circular arc $P\ C$, which is changed into the transition curve $A\ M\ C$.

Let $E\ P\ G$ be tangent to $P\ C$ at P and therefore parallel to $A\ C^1$, and make $C\ C^1$ perpendicular to $A\ C^1$.

Also in Fig. 14, let vertical heights represent degrees of curvature at any point and horizontal distances, measurements along the cubic parabola.

Then the rectangle D B will represent graphically the circular arc $P\ C$, and the triangle $A\ B\ C$ represent graphically the cubic parabola $A\ M\ C$, and from this diagram and Fig. 13 we may readily conclude:

(1) Because the total angles of the arc $P\ C$ and the transition $A\ M\ C$ are equal, therefore the area of the triangle $A\ B\ C$ must equal the area of the rectangle $D\ B$, and therefore a transition curve is always twice as long as that portion of the circular curve which it replaces.

(2) Because the triangles $A\ M\ N$ and $C\ D\ N$ are equal and similar, therefore the angular deflections or offsets from the tangent to every point in $A\ M$ (Fig. 13), and