∴ The ratio of x to y when x becomes indefinitely great is

$$\frac{x}{y}=-1, \text{or } \frac{x}{y}=0.$$

(This question being a biquadratic in y, that letter has four values for any given value of x; for that particular value of x characterized by $x=\infty$, the two real values of y are $y=-\infty$, and $y=0$.)

6. If y be a function of x, and if for consecutive values of x, y at first increases and then decreases, that value of x for which y ceases to increase and begins to decrease is a *maximum* solution. If y at first decreases and then increases, that value of x for which y ceases to decrease and begins to increase is a *minimum* solution.

Example—Let a be the number and x one of the parts;

Then $\dfrac{x(a-x)}{x^2+(a-x)^2}$ is the given function$=y$;

$$\text{Whence } x=\frac{2ay+a\pm\sqrt{(a^2-4a^2y^2)}}{2(1+2y)}.$$

Now on account of the radical y cannot be increased indefinitely without giving rise to an imaginary expression, and it therefore has a maximum value, which occurs when $a^2-4a^2y^2=0$, or $y=\frac{1}{2}$.

$$\text{Whence } x=\frac{a+a}{2(1+1)}=\frac{a}{2} \text{ for a maximum.}$$

∴ The number must be halved, and the quotient arising from dividing the product of the parts by the sum of the squares takes then its maximum value of one-half.

(This question should have read....."by the sum of their squares may be a *maximum or a minimum*." It could not however mislead, since it is by the solution itself that we determine algebraically which solution the question admits of.)

7. (1) The fundamental formulæ in $A.P$ are $S=\frac{n}{2}\{2a+(n-1.)d\}$, and $z=a+(n-1.)d$; from which we must eliminate n since z is to be expressed in terms of a, d, and s.

From the second $n=\dfrac{z-a}{d}+1$;

and this substituted in the first gives—

$$s=\frac{z-a+d}{2d}(2a+z-a);$$

whence by solution—

$$z=-\frac{d}{2}\pm\tfrac{1}{2}\sqrt{\{4a^2+(8s+4a+d)d\}}.$$

(2) As the nth term is *any* term we obtain the terms of the series by putting 1, 2, 3, &c. for n in the formula for the nth term.

Now nth term of first series$=\frac{1}{3}(n+2)$,
∴ First " " " $=1$,
and sec'd " " " $=1\frac{1}{3}$,
∴ $a=1$, and $d=\frac{1}{3}$.
Similarly for the second series, we find
$a_1=1$, and $d_1=\frac{3}{2}$.
Then for the sums of n terms

$$s=\frac{n}{2}\{2a+(n-1.)d\}=\frac{n}{2}\{2+\tfrac{1}{3}(n-1)\}.$$

$$s_1=\frac{n}{2}\{2a_1+(n-1)d_1\}=\frac{n}{2}\{2+\tfrac{3}{2}(n-1)\}.$$

and the second of these is to be four times the first;

∴ $2+\frac{3}{2}(n-1)=4\{2+\frac{1}{3}(n-1)\}$
from which $n=37=$ number of terms.

(3) $$\frac{s_1}{s}=\frac{2+\frac{3}{2}(n-1)}{2+\frac{1}{3}(n-1)}=\frac{3+9n}{10+2n};$$

a ratio which increases as n increases. Dividing by n numerator and denominator, and then making $n=\infty$ we obtain,

$$\frac{s_1}{s}=\tfrac{9}{2}=4\tfrac{1}{2} \text{ as the greatest ratio.}$$

8. Let a=earth's attraction, m=its mass, r=its radius, l=the length of a given pendulum at its surface, and n=the number of beats made per second by that pendulum; and let a_1, m_1, r_1, l_1, and n_1 denote like quantities with respect to the moon.

Since a varies as $\dfrac{m}{r^2}$, we may write $a=\dfrac{pm}{r^2}$, where p is an unknown but constant factor.

Similarly, $a_1=\dfrac{pm_1}{r_1^2}$;

∴ $$\frac{a}{a_1}=\frac{m}{m_1}\left(\frac{r_1}{r}\right)^2.$$

As l varies as $\dfrac{a}{n^2}$, we write $l=\dfrac{qa}{n^2}$.

Similarly $l_1=\dfrac{qa_1}{n_1^2}$;

∴ $$\frac{l_1}{l}=\frac{a_1}{a}\left(\frac{n}{n_1}\right)^2=\frac{m_1}{m}\left(\frac{r}{r_1}\right)^2\left(\frac{n}{n_1}\right)^2;$$

∴ $$l_1=l\cdot\frac{m_1}{m}\left(\frac{rn}{r_1n_1}\right)^2.$$

Now $l=6.26$, $m=75$, $r=4000$, $n=\frac{1}{2}$, $m_1=1$, $r_1=1100$, & $n_1=1$.

∴ $l_1=6.26\times\frac{1}{75}(\frac{20}{11})^2=6.9$ inches nearly.

9. Let nCr denote the combinations of n things taken r together.
Then A and B are together $13C_4$ times$=715$,
and A, B, & C " $12C_3=220$ times (1)
∴ A and B without C $=495$ " (2)
A, B, C, and D are together $11C_2$ times$=55$ times (4)
But A, B, and C are together 220 times;
∴ A, B, and C will be together when D is absent $220-55$ times $=165$ times.

Similarly A, B, and D will be together when C is absent 165 times; ∴ A and B will be together with C or D, but not with both, $2\times165=330$ times (3).

10. (1) (a) In this case the coefficient of x must be zero;

∴ $$\frac{a-b}{ab}=0, \quad \text{whence } a=b.$$

(β) In this case the expression must be a complete square, and hence $\left(\dfrac{a-b}{ab}\right)^2=\dfrac{4ab}{a-b}$,

∴ $$\left(\frac{a-b}{ab}\right)^3=4, \text{ or } \frac{1}{b}-\frac{1}{a}=\sqrt[3]{4},$$

∴ $$\frac{1}{b}=\sqrt[3]{4}+\frac{1}{a},$$

∴ $$b=\frac{1}{\sqrt[3]{4}+\frac{1}{a}}.$$

(2) Here $x_1x_2=\dfrac{ab}{a-b}$, and $x_1+x_2=-\dfrac{a-b}{ab}$;

∴ $$\frac{x_1+x_2}{x_1x_2}=\frac{1}{x_2}+\frac{1}{x_1}=-\frac{a-b}{ab}\cdot\frac{a-b}{ab}=-\left(\frac{a-b}{ab}\right)^2;$$

or $$\frac{1}{x_1}+\frac{1}{x_2}=-\left(\frac{1}{b}-\frac{1}{a}\right)^2.$$

[NOTE.—We return cordial thanks to our valued correspondent, and hope to hear from him (or her ?) again.—Math. Ed.]

CENTRE OF GRAVITY.

In working problems engaging the centres of gravity of the cone, its frustra, the hemisphere, and segment of the sphere, I found it necessary to investigate where these centres are. The following formulæ result from my calculations, and may be new to many, as they were to me.

For the cone:—If m stands for the height of frustrum, b the diameter of less end, a that of the greater end, then the centre of gravity is—

$$\frac{m}{4}\times\frac{b^2+2ab+3a^2}{a^2+ab+b^2}=\text{distance centre of gravity from } b.$$

When $b=0$, the solid becomes a cone, its centre of gravity is $\frac{3}{4}m$.

The centre of gravity for the *segment of a sphere is*

$$\frac{8rv-3v^2}{12r-4v}, \; r=\text{radius and } v=\text{height.}$$

When $v=r$, centre of gravity is

$$\frac{8r^2-3r^2}{8r}=\tfrac{5}{8}r, \text{ or } \tfrac{3}{8}r, \text{ reckoned from centre of sphere.}$$

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