and we may write—

$$\left.\begin{aligned}\tan^{-1}x+\tan^{-1}y&=\tan^{-1}\frac{x+y}{1-xy}\\ \tan^{-1}x-\tan^{-1}y&=\tan^{-1}\frac{x-y}{1+xy}\end{aligned}\right\}\cdots\cdots\cdots(41)$$

Ex. 1. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\tan^{-1}\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}=\tan^{-1}1=\frac{\pi}{4}$

Ex. 2. $2\tan^{-1}\frac{1}{5}=\tan^{-1}\dfrac{\frac{2}{5}}{1-\frac{1}{25}}=\tan^{-1}\frac{10}{24}=\tan^{-1}\frac{5}{12}.$

The meaning of example 1 is that if two angles be constructed such that the tangent of the one is $\frac{1}{2}$ and the tangent of the other is $\frac{1}{3}$, the two angles together will make up 45° or $\frac{1}{4}\pi$.

42. To sum $\sin^{-1}x+\sin^{-1}y$.

Let $\varphi=\sin^{-1}x$ and $\theta=\sin^{-1}y$

Then $x=\sin\varphi$, $y=\sin\theta$, $\sqrt{1-x^2}=\cos\varphi$, $\sqrt{1-y^2}=\cos\theta$.

But $\sin(\varphi+\theta)=\sin\varphi\cos\theta+\cos\varphi\sin\theta$

$$=x\sqrt{1-y^2}+y\sqrt{1-x^2}$$

$$\therefore\ \varphi+\theta=\sin^{-1}\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\}$$

or $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\}$

Similarly

$$\cos^{-1}x+\cos^{-1}y=\cos^{-1}\{xy-\sqrt{(1-x^2)(1-y^2)}\}\quad\cdots(42)$$