which vanishes, the second complex term differing from the first only in the sign of one factor, having $(c-a)$ instead of $(a-c)$.

Hence the former polynome is divisible by $a-b$, and by symmetry it is also divisible by $a-c$, by $a-d$, by $b-c$, by $b-d$, by $c-d$.

Again, $(a+b)^5+(c+d)^5$ is divisible by $(a+b)+(c+d)$; for, on putting $a+b=-(c+d)$, it becomes $\{-(c+d)\}^5+(c+d)^5$ which $=0$.

Similarly the other terms of the former of the given polnomes are each divisible by $a+b+c+d$, and consequently the whole is so divisible.

Now all these factors are different from each other, hence the former of the given polynomes is divisible by the product of these factors, *i.e.*, by the latter of the given polynomes.

Both of these polynomes are of seven dimensions, hence their quotient must be a number, the same for all values of a, b, c, d.

Put $a=2$, $b=1$, $c=0$, $d=-1$, and divide. The quotient will be found to be -5.

$\therefore\ \{(a+b)^5+(c+d)^5\}(a-b)(c-d)+\{(b+c)^5+(a+d)^5\}\times$
$(b-c)(a-d)+\{(b+d)^5+(c+a)^5\}(b-d)(c-a)=-5(a-b)(c-d)$
$\times(b-c)(a-d)(b-d)(c-a)(a+b+c+d)$.

N.B.—It is not always necessary to find the factors of the divisor, as the following examples show.

10. Prove that x^2+x+1 is a factor of $x^{14}+x^7+1$.

x^2+x+1 will be a factor of $x^{14}+x^7+1$ provided

$x^{14}+x^7+1=0$ if $x^2+x+1=0$.

$$\begin{aligned}
&\text{If } x^2+x+1 &&=0\\
&\therefore\ x^3+x^2+x &&=0\\
&\therefore\ x^3+x^2+x+1 &&=1\\
&\therefore\ x^3 &&=1\\
&\therefore\ x^6=1 \text{ and } x^{12} &&=1\\
&\therefore\ x^7=x \text{ and } x^{14} &&=x^2\\
&\therefore\ x^{14}+x^7+1 &&=x^2+x+1=0
\end{aligned}$$

$\therefore\ x^2+x+1$ is a factor of $x^{14}+x^7+1$.