7. Solve the simultaneous equations

$$\tfrac{1}{5}x+y+7z=41\,;\ x+\tfrac{2}{5}y+7z=42\,;\ x+y+z=15.$$

8. Reduce

$$\frac{x^4+a^2x^2+^4}{x^3-a^2}\times\frac{x+a}{x^2+ax+a^2}\div\frac{x^2-ax+a^3}{x+a}-\frac{x^3-a^3}{x^2+a^3}\times\frac{x^2ax+a^2}{x-a}$$

$$\times\frac{x+a}{x^2+ax+a^2}$$ to its simplest form.

9. A milkman has three cans of 10, 7, and 4 quarts respectively; the first is full, the other two are empty: he is required to divide the ten quarts into five quarts in the ten-quart can and five quarts in the seven-quart can. How will he do it?

10 Three numbers are in geometrical progression, the sum of the first and third exceeds the double of the second by unity; and if from the difference of the first and third one be taken, the result is one-third of the second. What are the numbers?

11. A agrees to pay B a total sum of £300, in three instalments of £100, at the end of one, two, and three years respectively. He fails to make any payments, and at the end of four years B demands payment. Reckoning compound interest at 4 per cent., how much should B receive?

12. Assuming, for the purpose of this question, that a full-rigged ship has 40 hands, a schooner 15, and a steamer 10; on a certain day 36 vessels, all either ships, schooners or steamers, arrived in port: they had, in all, 750 hands; the hands on board the ships would be just numerous enough to man all the schooners and twice as many st ners as arrived that day. How many of the vessels were ships, schooners, and steamers, respectively?

SOLUTIONS.

1. 1 metre.

2. ·0010000. Use abridged method.

3. ·1859244. Shorten work by following rule: in finding the square root of a number, when $n+1$ figures of the root have been obtained, n more may be obtained by simply dividing the last remainder by the last trial divisor. Thus in the present example 7 figures are required. The first four, ·1859, are found by the ordinary process; and then the division of the remainder 90800...... by the trial-divisor 37182, will give correctly three other digits, 244. See Colenso's Algebra, Pt. II., § 16.

4. $\frac{23}{130}$.

5. 1 gallon weighs 160 oz.; 1 cub. ft. weighs 1000 oz.; ∴ 1 gallon $=\frac{160}{1000}\times 1728$ cub. inches. Also 1 litre $=\left(\frac{315}{80}\right)^3$ cubic inches, from Ex. 1. Hence one gallon contains $\frac{160}{1000}\times 1728\times\left(\frac{16}{63}\right)^3=4{\cdot}53$ litres nearly.

6. Let x^2+cx+d be the factor; the product $(x^2+ax+b)\ (x^2+cx+d)$ must therefore be *identically* equal to x^4+px^3+...... Multiplying out, and equating coefficients of x we have

$$q=b+d+ac$$
$$p=a+c$$
$$r=ad+bc$$
$$s=bd.$$

From first two $c=p-a,\ d=q-b-a\ (p-a)$. Substituting in third and fourth $r=a^3-a^2p+a(q-2b)+bp,\ s=ab\ (a-p)-b\ (b-q)$.

7. $x=7=z=5$.

8. $1-1=0$.

9. The simplest solution of the indeterminate equation $4x+7y=5$, is $x=3, y=-1$. Hence, evidently, if the milkman can succeed in filling the four-quart can three times, and, from the milk so put in, can manage to fill the seven-quart can once, he will have accomplished the required division: the method of doing this is sufficiently straightforward.

10. Let x, y, z be the numbers. Then $x+z=2y+1,\ z-x-1=\frac{2}{3}y$; also $xz=y^2$. From the first two equations $6x=4y$, and $3z=4y+3$; substituting these values of x and z in the third equation we get $y=6$; thence $x=4, z=9$.

11. Amount $=100\{(1.04)^3+(1.04)^2+(1.04)\}=$ £324.6454.

12. Let x, y, z be the number of ships, schooners and steamers respectively. $x+y+z=36,\ 40x+15y+10z=750,\ 15y+20z=40x$. Whence $x=11,\ y=12,\ z=13$.

GEOMETRY.

TIME—THREE HOURS.

Examiners—Dr. J. Hopkinson, M.A., F.R.S., and Rev. Professor Townsend, M.A., F.R.S.

[Candidates are at liberty to use all intelligible abbreviations in writing out their answers.]

1. Two finite right lines, of any lengths, being supposed to radiate, in any directions, from a common terminal point; show that the angle they determine is equal to that determined by their two productions through the point.

2. Two rectilinear segments, of any lengths, being supposed to have a common middle point, but not a common direction; show, assuming the preceding property, that they are the two diagonals of a parallelogram.

3. Two triangles, having a common base, being supposed to have their two vertices on a common parallel to the base; show that the four parallelograms, on the same base, having their four sides for diagonals, are equal in area.

4. By aid of the preceding, or otherwise, construct, on a given base, a triangle of given area, having its vertex on a given indefinite right line not parallel to the base; and determine the number of solutions.

5. Two chords of a circle, intersecting at a point within the circumference, being supposed to make equal angles with the line connecting the point with the centre; show that the two segments of either are equal to the two segments of the other.

6. By aid of the preceding, or otherwise, construct an isosceles triangle of given vertical angle, having its vertex at a given point within a given circle, and both extremities of its base on the circumference of the circle; determine also the number of solutions.

7. A quadrilateral, of the ordinary form, being supposed inscribed in a circle; show that the sum of either pair of its opposite angles is equal to the sum of the other pair.

8. The quadrilateral, in the preceding property, being supposed to be a parallelogram; show, as a consequence from the property, or otherwise, that its two diagonals pass through the centre of the circle.

9. A quadrilateral, of the ordinary form, being supposed circumscribed about a circle, show that the sum of either pair of its opposite sides is equal to the sum of the other pair.

10. The quadrilateral, in the preceding property, being supposed to be a parallelogram; show, as a consequence, from the property, or otherwise, that its two diagonals pass through the centre of the circle.

11. Divide a given finite right line into two unequal segments, so that the rectangle contained by the whole line and the lesser segment shall be equal to the square of the greater segment.

12. By aid of the preceding or otherwise, construct, on a given base, an isosceles triangle, each of whose base angles shall be double of its vertical angle; and complete, by aid of it, the construction of a regular pentagon on the base.

[Examiners in Canada would do well to mark the custom indicated by the note at the commencement of this paper. The custom of varying the wording of familiar propositions is also an excellent one; the examiner has then some assurance that candidates will not recognize the propositions merely by the jingle of words.]

SOLUTIONS.

1. Prop. 15, Bk. I., Euclid.

2. Joining the extremities of the line, we find we are asked to prove the converse of the problem,—the diagonals of a parallelogram bisect each other.

3. Establish Prop. 35, Bk. I., Euclid, under which the problem is an example.

4. From the preceding it follows that triangles on the same base and between the same parallels are equal. On both sides the given base let there be triangles of the given area. Through their vertices draw lines parallel to the base; the points where they intersect the given line, when joined to the base, will give the triangles required, two in number.