the sides of a triangle ABC from the centre of the circumscribing circle, prove that

$$aqr + brp + cpq = \frac{abc}{4}.$$

XV. A circle is described through the foci of an ellipse and any point on its circumference. Two tangents are drawn to this circle through one extremity of the major axis. Shew that the locus of the points of contact of these tangents is a circle whose radius is equal to the minor axis of the ellipse.

16. CP, CD are conjugate semi-axes of an ellipse; PNE is drawn parallel to the minor axis CB, meeting the major axis in N and CD in E. Prove that the area of the triangle

$$PCE \text{ is } = 2\,CB^2.\,\frac{PN}{CN}.$$

17. OA and OB are asymptotes of a hyperbola; CEI a tangent perpendicular to OA; from C the foot of this perpendicular CD is drawn at right angles to OB. Prove that every perpendicular drawn from the curve to CD or CD produced will subtend at E, where the tangent CEI meets the hyperbola, a constant angle.

18. TP, TQ are two tangents to an ellipse at right angles to one another, S a focus, prove that

$$\sin^2 SPT + \sin^2 SQT = \text{constant}.$$

SOLUTIONS

To Natural Philosophy paper of London University (published in March No.), by Messrs. R. F. GREENLEES, J. EWINGS, C. MOREY, T. STEVENSON, Scholars, Collingwood Collegiate Institute:—

3. What is the centre of gravity of a body? A line is drawn across an equilateral triangle, of 12 inches side, parallel to its base, and so as to cut off one-fourth of its area. Find the distance of the base from the centre of gravity of the remainder.

For centre of gravity see Cherriman & Baker's Mechanics.

Since ABC, and AEF are similar triangles, and that AEF is $\frac{1}{4}$ ABC,

$$\therefore \text{(VI. 19) } AD = \frac{AG}{2},$$

and $AG = 6\sqrt{3}$, $\therefore$ $AD = 3\sqrt{3}$.

Let x = distance of centre of gravity of $EFCB$ from base.

Then taking moments about the base,

$$36\sqrt{3} \times 2\sqrt{3} = 9\sqrt{3}\,(3\sqrt{3} + \sqrt{3}) + 27\sqrt{3}\,x.$$

$$72\sqrt{3} = 36\sqrt{3} + 27x$$

$$x = \frac{36\sqrt{3}}{27} = \frac{4\sqrt{3}}{3} \text{ inches from base.}$$

4. Equal forces act for the same time upon bodies of different mass. What is the relation between the effects which they produce? Describe fully the unit of force implied in the equation $P = mf$.

See Cherriman & Baker's Mechanics.

5. A body is allowed to fall freely from rest. Find an expression for its velocity at any point in terms of the distance through which it has fallen and the acceleration of gravity. If $g = 981$ centimetre-second units, from what height must a body fall in order that it may have a velocity of 50 metres per second on striking the ground?

(1) See Cherriman & Baker's Mechanics.

$$v^2 = 2gs.$$

$$(50)^2 = 2 \times 9{\cdot}81\, s$$

$$s = \frac{50 \times 50}{2 \times 9{\cdot}81} = 127\tfrac{14}{981} \text{ metres.}$$

6. A cubic foot of water may be assumed to contain 1,000 ounces, while a gallon contains 10lbs. Two gallons of water are placed in a cylindrical can, 10 inches in diameter. Find the whole pressure upon the curved surface of the can.

Radius of base = 5, $\therefore$ area of base = $\frac{25}{144}\pi$ square feet.

2 gals. = 20 lbs. = 320 oz.;

$\therefore$ volume of water = $\frac{320}{1000}$ cb. ft. = $\frac{8}{25}$ cb. ft.

$\therefore$ Depth of water in cylinder = $\frac{8}{25} \div \frac{25}{144}\pi$;

$$= \frac{8}{25} \times \frac{144 \times 7}{25 \times 22} = \frac{56 \times 72}{11 \times (25)^2}.$$

$$\text{Circumference of base} = \frac{10 \times 22}{7};$$

$$\therefore \text{area of surface} = \frac{56 \times 72}{11 \times (25)^2} \times \frac{10 \times 22}{7} =$$

$$\frac{192}{125} \text{ square feet.}$$

$$\therefore \frac{192}{125} \times \frac{1000}{16} \times \frac{28 \times 72}{11 \times (25)^2} = \frac{193536}{6875} = 28\tfrac{1036}{6875}$$