function of $w^c$. In like manner $X_{c\delta}^{-\frac{v\delta}{n}}$ is a rational function of $w^c$, and so on. Therefore the second of equations (92) may be written

$$k_c = w^r (A_{cv} A_c^{-v}) QM_c (q_{c\sigma} q'_{c\tau} \ldots),$$

where $M_c$ is a rational function of $w^c$. In like manner, from the first of equations (92),

$$k_1 = w^a (A_v A_1^{-v}) QM_1 (q_\sigma q'_\tau \ldots),$$

$M_1$ being what $M_c$ becomes in passing from $w^c$ to $w$. By § 4 we can change $w$ in this last equation into $w^c$. This gives us

$$k_c = w^{ac} (A_{cv} A_c^{-v}) QM_c (q_{c\sigma} q'_{c\tau} \ldots).$$

Comparing this with the value of $k_c$ previously obtained, $w^r = w^{ac}$. Therefore the first of equations (87) becomes

$$R_{cv}^{\frac{1}{n}} = w^{ac} A_{cv} Q (\phi_{cv\sigma}^{\sigma} \psi_{cv\tau}^{\tau} \ldots)^{\frac{1}{n}}.$$

Replacing $Q$ by $(F_{cv\beta}^{\beta} X_{cv\delta}^{\delta} \ldots)^{\frac{1}{n}}$, and putting $e$ for $c$, which we are entitled to do because $w^e$ may be any one of the roots included under $w^c$,

$$R_{ev}^{\frac{1}{n}} = w^{ea} A_{ev} (\phi_{ev\sigma}^{\sigma} \psi_{ev\tau}^{\tau} \ldots F_{ev\beta}^{\beta})^{\frac{1}{n}},$$

which is the form of $R_{ev}^{\frac{1}{n}}$ in (77).

## *Sufficiency of the Forms.*

§ 45. Here we assume that $R_1$ has the form (72), and that the terms in (78) are determined by the equations (76), (77), etc., while $R_0^{\frac{1}{n}}$ receives its rational value. We have then to prove that the expression (73) is the root of a pure uni-serial Abelian equation of the $n^{\text{th}}$ degree, provided always that the equation of the $n^{\text{th}}$ degree, of which it is the root, is irreducible.

§ 46. *In the first place*, it has been shown that there is an $n^{\text{th}}$ root of $R_0$ which has a rational value; and, by hypothesis, $R_0^{\frac{1}{n}}$ has been taken with this rational value. *In the second place*, an equation of the type (3) subsists for every integral value of $z$. For, let $z$ be a multiple of $n$. In that case it may be taken to be zero. Then

$$(R_z R_1^{-z})^{\frac{1}{n}} = R_0^{\frac{1}{n}}. \qquad (93)$$

But $R_0$ is the $n^{\text{th}}$ power of a rational quantity. Therefore (93) is an equation of the type (3). If $z$ is not a multiple of $n$, it may be a multiple of some of the factors of $n$, say $b$, $d$, etc., though not of others, say $s$, $t$, etc. Because $z$ is a multiple of $b$, and $b\beta = n$, $z\beta$ is a multiple of $n$. Therefore $F_{z\beta} = F_0$. And $F_0^\beta$ is the $n^{\text{th}}$ power of a rational quantity. Therefore $F_{z\beta}^\beta$ is the $n^{\text{th}}$ power of a