

These expressions for p_4 and p_5 furnish the criterion of solvability for the quintic

$$x^5 + p_4x + p_5 = 0. \quad (41)$$

The root of the equation is

$$x = \theta^{\frac{1}{5}} + \alpha\theta^{\frac{2}{5}} + \lambda\alpha^2\theta^{\frac{3}{5}} - \lambda\alpha^3\theta^{\frac{4}{5}},$$

where λ is a root of the quartic equation

$$\lambda^4 - m\lambda^3 - 6\lambda^2 + m\lambda + 1 = 0,$$

and

$$\alpha = -\frac{\lambda^2 + 1}{n\lambda(\lambda - 1)},$$

and

$$\theta = -\frac{n^3\lambda(\lambda - 1)^3}{(16 + m^2)(\lambda + 1)(\lambda^2 + 1)}.$$

In the same issue of the *Journal* in which these results were established, Mr. J. C. Glashan of Ottawa, in "Notes on the Quintic," gave the relations between the coefficients of the solvable quintic

$$x^5 + p_5x^4 + p_4x^3 + p_3x^2 + p_2x + p_1 = 0,$$

and, in his wider formulæ, the forms of p_4 and p_5 in (40) are included. They were subsequently announced by Mr. Emory McClintock, who had discovered them independently. It is to be regretted that Mr. Glashan has not made public the method by which his conclusions were reached.

§25. From our present position the criterion of solvability of the quintic (41) can be at once deduced, and the solution of the equation effected more readily than by the process employed in the article of the *Journal* just referred to. For, put

$$m = \frac{4A}{y} \text{ and } n = 2t;$$

then the first of the equations (9) may be written

$$p_4 = 5y(3 - m). \quad (42)$$

Also, by the second of equations (9),

$$p_5 = -4B + 40ty. \quad (43)$$

But, from the first of equations (8), $B = -t(y + 2A)$. Therefore

$$p_5 = 44ty + 8At = 2ty(22 + m). \quad (44)$$

And, by (15), in connection with (14),

$$t^4y - y^3 = A^2 = \frac{m^2y^2}{16},$$

$$\therefore y = \frac{16t^4}{16 + m^2} = \frac{n^4}{16 + m^2}.$$