primitive and indefinable ideas of length and direction, and in these we have our whole idea of space; for space admits of all distances in all directions.

There is, however, one prominent difference between rotation, and length or transference; namely, that a rotating line, after making one complete revolution, returns to its first direction, and thereafter repeats all its directions in succession. But a point which, by translation, describes a straight line, will never return to its first position, at least as far as we can reason about it. Hence the rotation of a line has a period or cycle in which it goes through all its possible directions; but a linearly moving point has no cycle. On this account we have a natural unit of angle with which we can compare all other angles, while we have no natural unit of length, so that any unit of length must be arbitrary.

Now the simplest unit of angle that we can take is the angle contained in this cycle, or the angle described by a complete rotation of the describing line. This has been called a perigon, a round angle (rather a singular name), and a circumangle. I prefer the last.

Then we may define a straight angle as being one-half a circumangle, and a right angle as being one-fourth of a circumangle; and thus all straight angles are equal to each other and so also all right angles are equal to one another.

It is convenient and profitable, in proving geometrical relations to make use of motion, at least in imagination, in order to establish our results. Thus by the simple rotation of a line about the three vertices of a triangle, we can most easily show that the sum of the three interior angles is a straight angle.

Some people may object to this method. If they do, they must, to be logical, also object to the proof of Euc. I. 4, for that is effected by superposition, *i.e.*, by motion. And if any one objects, the burden is upon him to establish, if he can, a system of Geometry from which motion or superposition is excluded.

In Euclidian Geometry at any rate, that is, in the Geometry in which superposition is admissible, this latter process is the final court of appeal. We prove that two finite lines are equal when we show that their end points can, by superposition, be made to coincide; and we prove that two angles are equal when we show that we can so superimpose one of them upon the other that their arms may exactly coincide. And all our after work has reference to, and ultimately depends upon these two proofs of equality.

Any further consideration on this matter I must leave for some future time.

Let A B C be a triangle, in which the order of the vertices A, B, C is that of positive rotation, that is, in the contrary way to that in which the hands of a clock move.

Let the line P, Q lie along the side C, A, so that Q is beyond A, and let the end Q be marked with an arrow head. Rotate the line negatively about the point A until it comes to coincide with the side B, A. In this rotation the line describes the angle A. Next rotate the line P, Q about the point B, negatively, until it comes to coincide with the side B, C. In this second rotation the line describes the angle B.

Finally rotate P Q negatively about C until it comes to coincide with A B. The line has now described successively the three angles A, B and C, and it has exactly reversed its direction in doing so. Therefore the sum of the three angles is one-half a circumangle, or a straight-angle.