Solution by Prof. Edgar Frisby, M.A., Naval Observatory, Washington:—

105. If $ax^n = by^n = cz^n$, and if

$$x^{-1} + y^{-1} + z^{-1} = k^{-1}, \text{ then}$$

$$(ax^{n-1} + by^{n-1} + cz^{n-1})^{\frac{1}{n}} = (a^{\frac{1}{n}} + b^{\frac{1}{n}} + c^{\frac{1}{n}})\, k^{\frac{n-1}{n}}.$$

$$\frac{a}{x^{-n}} = \frac{b}{y^{-n}} = \frac{c}{z^{-n}} = \left(\frac{a^{\frac{1}{n}} + b^{\frac{1}{n}} + c^{\frac{1}{n}}}{x^{-1} + y^{-1} + z^{-1}}\right)^n =$$

$$\left\{\left(a^{\frac{1}{n}} + b^{\frac{1}{n}} + c^{\frac{1}{n}}\right) k\right\}^n = \frac{ax^{n-1}}{x^{-1}} = \frac{by^{n-1}}{y^{-1}} =$$

$$\frac{cz^{n-1}}{z^{-1}} = \frac{ax^{n-1} + by^{n-1} + cz^{n-1}}{x^{-1} + y^{-1} + z^{-1}} = \left(ax^{n-1} + by^{n-1} + cz^{n-1}\right)k$$

$$\therefore ax^{n-1} + by^{n-1} + cz^{n-1} = \left(a^{\frac{1}{n}} + b^{\frac{1}{n}} + c^{\frac{1}{n}}\right)^n k^{n-1} \text{ whence}$$

extracting the nth root we obtain the given result.

Solution by proposer, Angus MacMurchy, University College:—

Let ABC be a triangle; D, E, F, the points wherein its sides are touched by the inscribed circle; H, K, L, the feet of the perpendiculars from the vertices of the triangle DEF on the opposite sides; Δ, R, r, the area of triangle ABC, and the radii of its circumscribed and inscribed circles, and Δ_1 the area of triangle HKL, prove—

$$\Delta^{\frac{1}{2}} : \Delta_1^{\frac{1}{2}} = 2R : r.$$

The angles of triangle DEF and its pedal triangle HKL are—

$$\angle D = \tfrac{1}{2}(\pi - A), \text{ etc., etc.}$$

$$\angle H = \pi - 2D = A, \text{ etc., etc.}$$

The radius of circle round HKL, which is nine-point circle of triangle DEF, is $\frac{1}{2} r$. Now, triangles ABC, HKL, are similar, and have their linear dimensions proportional to the radii of their circumscribing circles.

$$\text{Therefore, } \Delta^{\frac{1}{2}} : \Delta_1^{\frac{1}{2}} = R : \tfrac{1}{2} r.$$

$$= 2R : r.$$

PROBLEMS.

132. Prove $\log. \sqrt[4]{x} = (x^{\frac{1}{4}} - 1)$

$$\frac{2}{x^{\frac{1}{8}} + 1} \cdot \frac{2}{x^{\frac{1}{16}} + 1} \cdot \frac{2}{x^{\frac{1}{32}} + 1} \ldots\ldots \text{ad infin.}$$

133. Prove $\dfrac{2^{r-1} - 1}{\underline{|r}} = \dfrac{1}{\underline{|r-1}\ \underline{|1}} +$

$$\frac{1}{\underline{|r-2}\ \underline{|2}} + \frac{1}{\underline{|r-3}\ \underline{|3}} + \ldots \text{ad infin.}$$

$$+ \frac{1}{2\left\{\underline{\left|\tfrac{r}{2}\right.}\right\}^2}; \text{ or, } \frac{1}{\underline{\left|\tfrac{r+1}{2}\right.}\ \underline{\left|\tfrac{r-1}{2}\right.}}$$

according as r is even or odd.

134. Prove $\dfrac{2^{2n} - 1}{\underline{|2n+1}} = \dfrac{1}{\underline{|2n-1}\ \underline{|2}} +$

$$\frac{1}{\underline{|2n-3}\ \underline{|4}} + \ldots\ldots\ldots + \frac{1}{\underline{|2n}}.$$

135. In a plane triangle ABC, if $\Sigma \sin^2 A = 2\sigma$, and $\sigma - \sin^2 A = \sigma_1$, etc., prove—

$$\sigma_2\,\sigma_3 + \sigma_3\,\sigma_1 + \sigma_1\,\sigma_2 =$$

$$\sin^2 A \sin^2 B \sin^2 C.$$

By W. J. Ellis, B.A., Mathematical Master, Collegiate Inst., Cobourg,—

136. Given that the centre of gravity of a hemisphere is $\frac{3}{8}$ of radius from base, find the centre of gravity of a hemispherical bowl whose internal radius is "m," and uniform thickness "p;" result to be given as distance from base in terms of m and p.

From this result find the distance of centre of gravity of a hemispherical surface from the centre of the base.