2. A man is supposed to be standing on the end of a car with a revolver in his hand; providing he shoots straight could he possibly shoot a man standing on the other end of the car, if it be going at the same rate as the bullet?

(1). Suppose the man with the revolver stands on the rear end of the car and shoots forward. Let the rate of the car be a miles an hour; then the bullet must also be moving at the rate of a miles an hour before the shooting takes place. If the bullet then leaves the revolver at the rate of a miles an hour it will then have a total velocity of $2a$ miles an hour, and will therefore overtake the man at the other end at the rate of a miles an hour; that is, at the same rate as when the car is not in motion.

(2). Suppose the positions reversed; then the velocity given to the bullet by the discharge is just sufficient to destroy the velocity it had on account of the motion of the car, and the bullet is consequently brought to rest; and since the victim is moving forward a miles an hour the bullet is approaching him at the same speed as before.

3. If two spheres of radii a and b touch each other internally, find the distance of the center of gravity of the solid contained between the two surfaces from the point of contact.

The volume of the inner sphere is

$$= \frac{4}{3} \times \frac{22}{7} \times b^3, \quad (1)$$

that of the outer $= \frac{4}{3} \times \frac{22}{7} \times a^3$, (2)

∴ volume of the solid between the surfaces

$$= \frac{4}{3} \times \frac{22}{7} \left(a^3 - b^3\right); \quad (3)$$

And if the weight of a unit of volume be taken as the unit of weight (1), (2) and (3) will represent the weights as well as the volumes.

Let x be the distance required.

Then the moment of (3) about the point of contact is

$$\frac{4}{3} \times \frac{22}{7} \left(a^3 - b^3\right) x, \quad (4)$$

And the moments of (1) and (2) about the same point are respectively

$$\frac{4}{3} \times \frac{22}{7} b^3 \times b, \quad (5)$$

$$\text{and } \frac{4}{3} \times \frac{22}{7} a^3 \times a. \quad (6)$$

and since the moment of the whole sphere about this point is equal to the sum of the moments of its two parts about the same point, therefore the sum of (4) and (5) is equal to (6).

$$\therefore (a^3 - b^3) x = a^4 - b^4$$

$$\therefore x = \frac{a^4 - b^4}{a^3 - b^3}.$$

4. If P be the continued product of n quantities in Geometric progression, S their sum and R the sum of their reciprocals, show that

$$P^2 = \left(\frac{S}{R}\right)^n$$

$$\text{Let } S = a + ar + ar^2 + \ldots.$$

$$\text{Then } R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots.$$

$$= \frac{1}{a^2 r^{n-1}} \left(a + ar + ar^2 + \ldots\right)$$

$$= \frac{1}{a^2 r^{n-1}} S$$

$$\therefore \frac{S}{R} = a^2 r^{n-1}$$

$$\therefore \left(\frac{S}{R}\right)^n = a^{2n} r^{n(n-1)}$$

$$P = a \times ar \times ar^2 \times \ldots.$$

$$= a^n r^{1+2+3+\ldots+n-1}$$

$$= a^n r^{\frac{1}{2} n (n-1)}$$

$$\therefore P^2 = a^{2n} r^{n[n-1]}$$

$$\therefore P^2 = \left(\frac{S}{R}\right)^n$$

5. If $a^3 + 8c = 4ab$ and $a^2 d = c^2$ show that $4bcd = 8ad^2 + c^3$

since $a^2 d = c^2$

$\therefore a^4 d^2 = c^4 = a^2 dc^2$ (1)

(The square of each of two equal quantities is equal to their product)

∴ multiplying the terms of

$4ab = 8c + a$