ARITHMETIC (FIRST CLASS).

1. Prove the rule for multiplying one fraction by another, and deduce that for dividing one fraction by another.

Prove

$$\frac{\lfloor 48}{\lfloor 13 \times \lfloor 35} + 3\cdot\frac{\lfloor 48}{\lfloor 12 \times \lfloor 36} + 3\cdot\frac{\lfloor 48}{\lfloor 11 \times \lfloor 37} + \frac{\lfloor 48}{\lfloor 10 \times \lfloor 38} = \frac{\lfloor 51}{\lfloor 13 \times \lfloor 38}.$$

Book work.

Bringing the fractions to a common denominator, and adding, we have

$$\frac{\lfloor 48}{\lfloor 13 \lfloor 38}\Big\{36.37.38+3.13.37.38+3.12.13.38+11.12.13\Big\} = \frac{\lfloor 51}{\lfloor 13 \lfloor 38}$$

2. Shew, without algebra, the reasons of the rules for pointing in multiplication and division of decimals.

Reduce to a decimal of four places:

$$\frac{1}{2^2}+\frac{2}{2^3}+\frac{3}{2^4}+\frac{4}{2^5}+\frac{5}{2^6}+\frac{6}{2^7}+\frac{7}{2^8}+\frac{8}{2^9}.$$

Book work.

Let $S=\frac{1}{2^2}+\frac{2}{2^3}+..+\frac{8}{2^9}$,

$$\therefore \frac{S}{2}=\frac{1}{2^3}+..+\frac{7}{2^9}+\frac{8}{2^{10}};$$

subtracting $\frac{S}{2}=\frac{\frac{1}{2^2}\left(1-\frac{1}{2^8}\right)}{\frac{1}{2}}-\frac{8}{2^{10}}$,

$$\therefore S=\frac{255}{256}-\frac{8}{512},$$

$$=.98044+..$$

3. A rectangular piece of ground contains 9 acres 1 rood 16⅓ poles; its length is to its breadth as 3 to 1: find (1) the distance round it, (2) the distance from one corner to the opposite corner.

$$9 \text{ ac. } 1 \text{ ro. } 16\tfrac{1}{3} \text{ po.} = \frac{4489}{3} \text{ po.} = \frac{\text{length}^2}{3},$$

$\therefore$ distance round=178⅔ po., and distance from corner to corner=70.62+.. poles.

4. Investigate a rule for finding the amount of an annuity at compound interest for a term of years.

I borrowed $2000 for four years at 10 per cent. compound interest, to be paid in four equal annual payments. Find the annual payment.

Book work.

Let $a$ be the annual payment,

$$\therefore a\{1+1.1+(1.1)^2+(1.1)^3\}=2000$$

$$a=\$430.941+..$$

5. A piece of glass whose specific gravity is 2.4, and whose weight is 4½ lbs., is found to weigh only 2¼ lbs. when weighed in a certain liquid. Find the specific gravity of the liquid.

If $v$ be volume of piece of glass, $e$ sp. gr. of liquid, $g$ local acceleration due to action of gravity,

$$\left.\begin{aligned} 2.25&=gev: \\ 4.5&=gv(2.4); \end{aligned}\right\} \therefore e=1.2.$$

6. Shew how to find the true discount for a given time and rate.

I bought a bill of goods amounting to $1040, for which I gave my note payable in six months without interest, and immediately sold the goods for $1200 on such a term of credit as made my gain 17 %, reckoning money worth 8 %. Find the term of credit.

Book work.

Gain by buying at credit=4% of $1040=$41.60; gain by selling=$160, but total gain=17½% of $1040=$182.

$\therefore$ $(160+41.60-182)=interest of $1200 for certain time.

$\therefore$ time=4.9 months.

7. Prove that the area of a circle = $\pi r^2$, or = radius × ½ circumference.

What is the proportionate error in the following rough rule for finding the area of a circle?—Take $\frac{7}{9}$ of the square on the diameter, and add one per cent.