whan $n=1$, they become

$$(x-y)(ab+xy),\ (a-b)(ab+xy)$$

and when $n=2$, we get

$$(x-y)(ab\,\overline{x+y} - xy\,\overline{a+b})$$

$$(a-b)(xy\,\overline{a+b} - ab\,\overline{x+y})$$

6. Let $\frac{a}{b} = \frac{c}{d} = x$

$$\therefore a = bx,\ c = dx.$$

then substitute these values for a, c, and the equality is established.

7. Let $\frac{a}{b} = x$, $\frac{c}{d} < x$

$$\therefore \begin{array}{l} a = bx \\ c < dx \end{array} \quad (1)$$

$$\therefore a + c < (b + d)\,x$$

$$\therefore \frac{a+c}{b+d} < x \therefore < \frac{a}{b}$$

similarly $\frac{a+c}{b+d}$ can be proved to be greater than $\frac{c}{d}$

again, from (1) $ac < bdx^2$

$$\therefore \frac{ac}{bd} < x^2$$

$$\therefore \text{sq. rt. of } \frac{ac}{bd} < x \therefore < \frac{a}{b}$$

and similarly it may be proved $> \frac{c}{d}$

$$\frac{a+c}{b+d} > \frac{\sqrt{ac}}{\sqrt{bd}}$$

$$\text{if } \frac{(a+c)^2}{ac} > \frac{(b+d)^2}{bd}$$

$$\text{if } \frac{a}{c} + \frac{c}{a} > \frac{b}{d} + \frac{d}{b}$$

$$\text{if } \frac{a}{c} - \frac{b}{d} > \frac{d}{b} - \frac{c}{d}$$

$$\text{if } \frac{a}{c} - \frac{b}{d} > \frac{\frac{a}{c} - \frac{b}{d}}{\frac{ab}{cd}} \quad (1)$$

$$\text{if } 1 > \frac{cd}{ab} \text{ and } \frac{a}{c} > \frac{b}{d}$$

$$\text{if } \frac{a}{c} > \frac{d}{b} \text{ and } \frac{a}{c} > \frac{b}{d}$$

or (1) holds if

$$\frac{a}{c} < \frac{b}{d} \text{ and } 1 < \frac{cd}{ab}$$

$$\text{if } \frac{a}{c} < \frac{b}{d} \text{ and } \frac{a}{c} < \frac{d}{b}$$

i. e. if $\frac{a}{c}$ does not lie between

$$\frac{b}{d} \text{ and } \frac{d}{b}$$

8. Let the last of these ratios $= r$
then the next to the last $= r^2$
" " " this $= r^4$, &c.
$\therefore$ the first ratio $=$ last ratio raised to the power 2^{n-1}.

9. Let $a = m\sqrt{b}$, and $c^2 = n_2 b_3$.
$\therefore a^2 = m^2 b$. $\therefore a^2 c^2 = m^2 n^2 b^4$.
$\therefore ac = mnb^2$.
$\therefore ac$ varies as b^2.

10. 1, 1.

11. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$\therefore$ in order that these values of x may be positive integers, $b^2 - 4ac$ must be a complete square, z^2 suppose,

$$\therefore x = -\frac{b \pm z}{2a}$$

$\therefore$ also $b + z$ and $b - z$ must be multiples of $2a$ and b and a must have different signs.

13. $$\frac{(H - a)(H - b)}{G^2}$$

$$= \frac{H^2 - \overline{a+b}\,H + ab}{G^2}$$

$$= \frac{H^2}{G_2} - \frac{\overline{a+b}\,H}{G^2} + \frac{ab}{G^2}$$

$$= \frac{H}{A} - \frac{a+b}{A} + 1, \text{ since } G_2 = AH.$$