command, and the vigorous vocal efforts, although annoying to quiet and sedentary people, expand the lungs, circulate the blood, and give it additional purity and nourishment, as the happy child unconsciously inhales great draughts of pure air in his shouting efforts. Mariners, who have to raise their voices higher than the stormy winds and louder than the roaring of the tempest, have generally expansive chests and voices of the deepest tone and greatest power. There is no fear of energetic speaking and shouting injuring the voice if the exercise be carried on with due regard to the physical organization. Hence the importance of making such exercises a part of the gymnastic training of youth.

These vocal exercises are simple and agreeable, and are of two classes. The first consists of a series of well-arranged breathing practices, which, as they fill and expand the whole breathing apparatus, are best conducted in the open air, and should at all times be made where there is an abundance of pure air. The pupils stand erect, with the shoulders thrown back and the chest fully expanded. The air is then inhaled through the nostrils until the lungs are filled to their fullest extent. Then the breath is sent out in various forms of practice. It is allowed to pass out calmly and without effort. It is expelled with a gentle force. It is driven out with great energy, as if the vocal gymnast would knock a house down with the expulsive action. Again it is slowly poured forth until entirely expelled, or it is sent out in rapid jets like the panting of a steam engine in full blast. The moment the breath is exhausted, the lungs are instantly and rapidly refilled, and thus every cell, the most remote in the lungs, which in our methods of reading and conversing we never use, is brought into full and vigorous play. The certain result of such exercises is to enlarge the lungs and the chest, and students who have faithfully carried out the practice have gained several inches in lung capacity in two or three months. I need scarcely say that such exercises, like all other gymnastics, must be in harmony with the physical organization. Excess in vocal as in limb gymnastics cannot fail to be injurious. But nature, in this as in all other bodily exercises, is a sure guide. Excesses are always followed by pain, while judicious and moderate practice leaves agreeable and healthful sensations.

The breathing exercises are a preparation for the voice exercises. Unless there be some organic defect every human being is created with the elements of a good voice, and if the natural powers were systematically cultivated, good voices in adult age would be the rule and not the exception. The expressiveness of speech which marks the delivery of a good reader or speaker is due to the management of the voice more than to the control of the intellect. It is true that to give the just expression, the voice must act under the guidance of the intellect—the reader must *understand* what he reads. But often the most intelligent and cultivated reader fails utterly in the expressive delivery of a passage which he thoroughly understands. He has no control over his voice, and he has no conception of the music of speech. This no doubt will be the great difficulty which we shall have to meet in introducing the subject into our public schools. But it is not greater than the difficulty of introducing musical culture into school education, and as we advance in musical culture the difficulties of developing the speaking voice will disappear. Much of the practice for improving and training the voice for singing is similar to the practice for drilling the speaking voice. It has, however, certain marked differences. The singing voice advances by sustained tones and incessant changes of modulation or pitch. The speaking tones must always be sustained; but while each tone of the singing voice is level and unvarying in pitch, each tone of the speaking voice invariably *ends* by *sliding upwards* or *downwards*. Now it is these slides—technically, inflections—which form one of the essential qualities of good reading; and while acuteness of ear to distinguish such variations is as necessary to the reader as the vocalist, the vocal action is different and demands a different mode of practice.

*(To be continued.)*

# Mathematical Department.

Communications intended for this part of the JOURNAL should be on separate sheets, written on only one side, and properly paged to prevent mistakes.
ALFRED BAKER, B.A., EDITOR.

## BINOMIAL THEOREM.

(Communicated by J. Morrison, M.D., M.A., Member of the Medical Council, and Examiner in Chemistry in the College of Physicians and Surgeons of Ontario.)

Lemma. If $n$ be a positive integer

$$\frac{a^n - b^n}{a-b} = a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots \text{ n terms.}$$

Hence the limiting value of the fraction $\frac{a^n - b^n}{a-b}$, when $a = b$, is $na^{n-1}$.

*To expand* $(1 + x)^n$, *n being a positive integer.*

Assume $(1+x)^n = 1 + Ax + Bx^2 + Cx^3 + \ldots$ (1)

when $A, B, \ldots$ involve $n$, but not $x$. By putting $x = 0$, we see the first term of the expansion must be 1.

Similarly, $(1+y)^n = 1 + Ay + By^2 + Cy^3 + \ldots$

Subtracting

$(1+x)^n - (1+y)^n = A(x \quad y) + B(x^2 - y^2) + \ldots$

Since $(1+x) - (1+y) = x - y$, we have by division

$$\frac{(1+x)^n - (1+y)^n}{(1+x) - (1+y)} = A + B.\frac{x^2 - y^2}{x-y} + C\frac{x^3 - y^3}{x-y} + \ldots$$

Now since this equation is true for all values of $x$ and $y$, it must be true when $x = y$, and then by the above lemma it becomes

$$n(1+x)^{n-1} = A + 2Bx + 3Cx^2 + \ldots$$

Multiply this by $1 + x$, and we have

$$n(1+x)^n = A + 2Bx + 3Cx^2 + \ldots \quad (2)$$
$$+ Ax + 2Bx^2 + \ldots$$

And (1) multiplied by $n$ gives

$$n(1+x)^n = n + nAx + nBx^2 + \ldots \quad (3)$$

And equating the coefficients of corresponding powers of $x$,

$$A = n.$$

$$2B + A = nA, \qquad \text{or } B = \frac{n-1}{2}.A.$$

$$3C + 2B = nB, \qquad \text{or } C = \frac{n-2}{3}.B.$$

&c. &c.

In which the law of the equations is evident. Substituting in each of these values that of the preceding coefficient, they become

$$A = n;\ B = n.\frac{n-1}{2};\ C = n.\frac{n-1}{2}.\frac{n-2}{3};\ \&c.$$

Substituting these values in (1), we have

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\underline{|2}}x^2 + \ldots$$

$$Cor.\ (a+x)_n = a^n\left(1 + \frac{x}{a}\right)^n = a^n\left\{1 + n.\frac{x}{a} + \frac{n(n-1)}{\underline{|2}}\left(\frac{x}{a}\right)^2 + \ldots\right\}$$

$$= a_n + na^{n-1}x + \frac{n(n-1)}{\underline{|2}}a^{n-2}x^2 + \ldots$$

If the indices be fractional or negative the lemma may be established as follows: