UNIVERSITY WORK.

MATHEMATICS.

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SELECTED PROBLEMS.

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1. If $a+b+c=0$, prove

$$\left(\frac{b^3-c^3}{a^3}+\frac{c^3-a^3}{b^3}+\frac{a^3-b^3}{c^3}\right)$$
$$\left(\frac{a^3}{b^3-c^3}+\frac{b^3}{c^3-a^3}+\frac{c^3}{a^3-b^3}\right)=$$
$$36-4(a^3+b^3+c^3)(a^{-3}+b^{-3}+c^{-3}).$$

2. If $(z+x-y)(x+y-z)=ayz$
$(x+y-z)(y+z-x)=bzx$
$(y+z-x)(z+x-y)=cxy$

prove $(abc)^{\frac{1}{2}}+a+b+c=4$.

3. Solve the equations
$(z+x-y)(x+y-z)=ax$
$(x+y-z)(y+z-x)=by$
$(y+z-x)(z+x-y)=cz.$

4. Solve the equations
$yz(y+z-x)=a$
$zx(z+x-y)=b$
$xy(x+y-z)=c.$

5. Prove that if
$ax+cy:by+dz=ay+cz:bz+dx=$
$az+cx:bx+dy,$
then each ratio $=a+c:b+d$;
prove also that $x^3+y^3+z^3=3xyz.$

6. Show that the sum of n terms of the series $1+5+13+29+61+\ldots\ldots$ is $4(2^n-1)-3n$.

SELECTED.

Solutions by Wilbur Grant, Collegiate Institute, Toronto.

(See MONTHLY for March, 1883.

1. $y=$ rate B travels,
$y+2.=$ " A "

$7y+7(y+2)=$ distance between towns.
$9(y+1)+9\frac{y+2}{2}=$ " " "

$7y+7(y+2)\quad 9(y+1)+9\frac{y+2}{2}\ y=8,$

distance $=126$ miles.

3. Let $x^2+mx+x=0$, $be=n$
whose roots are $\alpha+\beta$ and $\alpha\beta$
then $\alpha+\beta+\alpha\beta=-m$
$\alpha\beta(\alpha+\beta)=n$
but $\alpha+\beta=-p$
$\alpha\beta=q$
$\therefore -m=-(p-q)$
$n=-pq$
$\therefore=n$ is $x^2+(p-q)x-pq=0.$

4. General expression will be

$$S=\frac{n}{2}\left\{2a+\overline{n-1}cd\right\}$$

S of odd terms $=$

$$\frac{n+1}{4}\left\{2a+\left(\frac{n+1}{2}-1\right)2cd\right\}$$

S of even terms $=$

$$\frac{n-1}{4}\left\{2a+2cd+\left(\frac{n-1}{2}-1\right)2cd\right\}$$

$\therefore$ difference $=a+\frac{n-1}{2}cd.$

5. Consider the plank in equilibrium about the edge of the bench on the side on which the weights are placed.

Let $w=$ weight of plank, $a=$ length of plank, $x=$ distance from the weight to edge of bench in first case: taking moments about this edge.

$$\text{I. } w\left(\frac{a}{2}-x\right)=200\times x.$$

$$\text{II. } w\left(\frac{a}{2}-\overline{x+2}\right)=120\times(x+2).$$

$$\text{III. } w\left(\frac{a}{2}-\overline{x+6}\right)=60(x+6).$$

Eliminating a and x we find $w=24$ lbs.

6. Let $r=$ radius of each bullet,
$a=$ specific gravity of 1st bullet,