which has neither greater nor less cogency and application to human life than geometrical theorems to the material world. In the language of the mathematician, physical measurement and geometrical are mutually asymptotic.

This distinction, which is of importance for our purpose, and frequently misapprehended, may become still clearer if we reflect what could have been the progress of physical science —in which advance appears, from one aspect, to lie ultimately in the possibility of measuring to extra decimal places (note the discovery of argon)—had geometry remained empirical. Imagine a stone geometry, in which deductions are made in terms of such points. lines, and surfaces as can be obtained on stone, with the aid of stone. How could such a geometry cope with the niceties of measurements flowing from the use of steel instruments on steel surfaces? Clearly we should need to reconstruct and refine our geometry incessantly, as instruments become more precise and muscles more adaptable. Stone geometry would succeed wooden, steel geometry stone, and soon we might be floundering in the difficulties of a celluloid geometry.

All this may appear trivial, but, in view of notorious historical misapprehension of the basis of scientific geometry, the grotesque misapplication of Euclid to elementary education, and the vagueness evinced by even well-educated people concerning the nature of geometrical truth, I believe such illustrations have their use. Moreover, it is high time that teachers turned their attention to the history and philosophy of the subject they teach.

To return to the measurement of the rectangular surface, our scientific geometrician has, we suppose, logically deduced from his conceptions of straight lines and rectangles a formula for obtaining the area of any rectangle whatsoever—*i.e.*, a rectangle in his ideal sense of the word. Then, with the utmost precision of which he is capable, he measures the lengths of two adjacent sides of the given material rectangular surface, and, according to his formula, multiplies together these numbers, thus obtaining, in units of area, the magnitude of the given rectangle. As far as his measuring precision is reliable, so far can he trust his result; the applicability and validity of his abstract formula he never dreams of questioning—and rightly.

Observe the difference between the two methods of procedure. In the first (the practical geometrician's method), we start with direct, particular sense-perception and experiment, and end with a wide empirical induction, based on repeated rough measurements; in the other the process starts with a general scientific conception (formula based on rigorous reasoning from definitions, etc.), and we end in getting, through its aid, a particular experimental result. One process leads to an experimental or empirical geometry; the other proceeds from a scientific geometry. One deals with particular facts; the other with general theorems.

I have stated above that the earliest documents—the Egyptian Rhind Papyrus—respecting the geometrical knowledge of the ancients consist of the statement of the results of particular measurements, or at most of empirically discovered rules. "The papyrus contains," says Allman ("Greek Geometry from Thales to Euclid") "a complete applied mathematics, in which the measurement of figures and solids plays the principal part; there are no theorems properly so called; everything is stated in the form of problems, not in general terms, but in distinct numbers—*e.g.*, to measure a rectangle, the sides of which contain two and ten units of length; to find the surface of a circular area whose