evident that the first arc must intersect the arc rG between the points g′ and C, and the other arc must intersect the arc nG, between the points g and G. In the same manner describe other arcs *in infinitum*, through each point of bisection of the distance of the last bisection from G on LG and KG—and the points B and D′—it is evident that the ultimate arc must pass through the point G, and be the arc BGD′—also that the arcs g′G and gG, must continually be diminished and equally vanish or become nothing—but the arcs n′n and n″n also must equally vanish, when the arcs g′G and gG vanishes—for when the arc xg′ coincides with the arc nG, and qg, coincides with rG, the arc sn′ must coincide with the arc mn, and the arc on″ must coincide with the arc pn—and the point n must be on the ultimate arc BGD′—but by construction Bp is equal to Bm—consequently the points BnGn′D′, are on the arc of a circle—and every point of intersection described between the point B and G, and D′ and G through points in the same manner equally distant from B and D′, must be on an arc of the same circle.

Or let the point G move along the circular arc BG, then the points n and r would equally meet in the point B, for by construction Bn and Br are equal to one another, and the arcs rG and nG, Br and Bn would equally vanish—consequently their ultimate ratios are equal*—therefore the arc Bm must be equal to the arc Bp—and the intersection n must be on the circular arc BGD′.

LEMMA 3.

FIG. 5. Let AC be equal to one half of the perimeter of the circumscribing triangle abd of the given circle ABD, Fig. 4, and from the point C as a center, describe the semicircle ABDE and make the chords AB and BD, each equal to the radius AC—next make the chord or distance A1, equal to the perimeter of the inscribed square efgh,—the distance A2, equal to the perimeter of the inscribed polygon of eight sides—the distance A3, equal to the perimeter of the polygon of sixteen sides, &c., of the circle ABC, Fig. 4,—also make A1′, equal to the perimeter of the circumscribing square e′f′g′h′, and A2′ equal to the perimeter of the polygon of eight sides—A3′ equal to the perimeter of the polygon of thirty-two sides, &c., hence the points 1, 2, 3, are the first three of the infinite series of the inscribed-polygonal perimeters A1, A2, A3, &c., and the points 1′, 2′, 3′, 4′, are the first four of the infinite series of the circumscribing polygonal perimeters of the given circle ABD Fig. 4,—and let it be granted the distance An, is equal to the perimeter of the polygon of an infinite number of sides, or is equal to the circumference of the given circle ABD, Fig. 4.

Again from the point B as a center, with the distances B1′, B 2′, B3′, &c., and from the point D, as a center, with the distance D1, D2, D3, &c., describe the intersections l, a, b, c, &c., and through the points l, a, b, c, &c., draw the line labc .. n,—by construction the points l, a, b, c, .. n, are on a curved line concave to the arc nD, and meeting the arc BD in the point n. Also from the point B as a center, with the distances B2′, B3′, B4′, &c.—and from D as center with the disances D1, D2, D3, &c., describe the intersections a′, b′, c′, &c., and draw through the points a′, b′, c′, ... n the line a′b′c′ .... n—by construction the points a′, b′, d′, ... n are on a curved line, concave to the arc nB and shall meet the arc BD in the point n.

For as the distances of the points 1, 2, 3, &c., and 2′, 3′, 4′, &c., from A, of each series approach nearer to the distance An, the nearer must the intersections approach on the curve line a′b′c′, &c., to the point n—hence the ultimate intersection must become infinitely near to, or coincide with the point n—consequently the line through the intersections a′, b′, c′, &c., must meet the arc BD in the point n.

LEMMA 4.

FIG. 5. Let it be granted that the curve lines abc ... n and a′b′c′ ... n, are drawn through an infinite number of intersections, and suppose the chords an and a′n drawn—and that the points 1 and 2, are variable ;—and that the point 1 moves or approaches towards

* Newton's Mathematical Principles of Natural Philosophy, Book 1, Section 1.