Appendix A: Allocation of Cheating and Inspection Resources

This Appendix provides technical details to support the analysis and conclusions in the text concerning the allocation direction of this research project. The text contains an outline of the model, in which the allocation of violations and inspections is treated as a constant-sum game between the players, Inspectee (E) and Inspector (R). E's expected utility (payoff) when there are $n > 0$ time periods (inspection opportunities) and R has k inspections ($0 \leq k \leq n$) is denoted $V_{n,k}$.

Suppose that R has k inspections remaining. R's (current) strategic variable, denoted $p = p_{n,k}$, is the probability that R will inspect during this time period. E's (current) strategic variable, $q = q_{n,k}$, is the level of violation during this time period. Both variables are restricted: $0 \leq p \leq 1$ and $0 \leq q \leq 1$. The parameters are the detectability $r \leq 1$ and the penalty $K > 0$. The conditional probability that a violation at level q will be detected, given that an inspection is carried out, is rq. E's expected payoff is increased by q units if no violation is detected, and decreased by K units if a violation is detected. All of these assumptions are embodied in the iteration equation

$$V_{n,k} = p[rq(-K) + (1-rq)(q) + V_{n-1,k-1}] + (1-p)[q+V_{n-1,k}] \qquad \text{(A1)}$$

which applies for $0 < k < n$. Thus, to find $V_{n,k}$ using (A1), first find $V_{n-1,k-1}$ and $V_{n-1,k}$, then find the optimal choices of p and q , and then apply (A1).

In order to use (A1) to find values $V_{n,k}$ recursively, some boundary values must first be determined. First suppose that $n > 1$ and $k = 0$. No inspections are available, so E's value is

$$V_{n,0} = q + V_{n-1,0}$$

Obviously, E maximizes $V_{n,0}$ by choosing $q = 1$. Because $V_{1,0} = 1$, iteration yields

$$V_{n,0} = n$$

Now suppose that $n = k \geq 1$. Then R inspects at every time period, so that

$$V_{n,n} = rq(-K) + (1-rq)q + V_{n-1,n-1} \qquad \text{(A2)}$$

Using calculus it is easy to show that the value of q maximizing $V_{n,n}$ is

$$q^* = \begin{cases} \dfrac{1-rK}{2r} & \text{if } K < 1/r \\ 0 & \text{if } K \geq 1/r \end{cases}$$