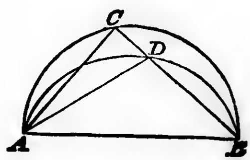
We here insert Euclid's proofs of Props. 23, 24 of Book III. first observing that he gives the following definition of similar segments :—

DEF. *Similar segments of circles are those in which the angles are equal, or which contain equal angles.*

PROPOSITION XXIII. THEOREM.

Upon the same straight line, and upon the same side of it, there cannot be two similar segments of circles, not coinciding with each other.



If it be possible, on the same base AB, and on the same side of it, let there be two similar segments of ⊙s, ABC, ABD, which do not coincide.

Because ⊙ ADB cuts ⊙ ACB in pts. A and B, they cannot cut one another in any other pt., and ∴ one of the segments must fall within the other. III (10)

Let ADB fall within ACB.

Draw the st. line BDC and join CA, DA.

Then ∵ segment ADB is similar to segment ACB,

$\therefore \angle ADB = \angle ACB$.

Or the extr. $\angle$ of a $\triangle$ = the intr. and opposite $\angle$, which is impossible ; I (16)

∴ the segments cannot but coincide.

Q. E. D.