Notes on Mathematics—No. III.

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In the October number, there were a few suggestions as to the teaching of Geometry. In addition, there was a fallacious example given in which *any* two lines are proved equal. A few have written, asking further concerning this problem. It may be proved *geometrically* that one of the perpendiculars 56, 67, falls inside the triangle, and one outside. It is easy enough to recognize this in the *construction*, but the *geometric* proof is not so easy. In order that the elementary, as well as the most advanced, may have a chance, we append another fallacy in which the reasoning is very simple. Euclid wrote one book containing only such examples. However, this book is lost, and this is reputed to be one of the theorems that the old Greek used in teaching Geometry. It may awaken thought in pupils of to-day.

The provincial examiner will tell you that a large percentage of the papers show an utter lack of any idea of a logical conclusion. In Problem I of the list below, how many will be able to say more than "I don't know"? They ought to be able to say decisively, "There is no conclusion." By a little change of figure, Problem II can be made to deceive even the most advanced high school pupils, unless they have learned to reason logically and carefully. A scheme used by some of the most successful teachers in Germany, is to get the pupils to prove a proposition without a figure. The reasoning will then appear the chief thing. Another method used with excellent results is that of asking the theorems that are used in getting from one to another, as suggested in Problem IV. More geometry can be taught in one hour by using these methods occasionally, than in a day using old routine methods that make no impression. In the list of problems below there is merely a skeleton of a plan that may be worked up into something useful in all the grades.

I. If AB > CD and CD < EF, what follows?

II. ABC is an isosceles triangle, with vertex at A. Draw *any* line AX to the base, and in the triangles ABX, ACX, we have three parts of one equal to three parts of the other. What proposition proves them equal in all respects?

III. What are the propositions in the first book that use the paralled axim directly.

IV. Trace the connection by giving all *necessary* intermediate theorems between I. 16 and I. 32. Between I. 4 and I. 24.

V. How many degrees in the interior angles of an octagon? If one angle is re-entrant, will it increase or decrease the number of degrees?

VI. Why cannot the propositions of Book III be put before those of Book II? Why not immediately after I. 10?

VII. Can you trisect *any* given angle?

VIII. Are the two circumferences of concentric circles parallel lines?

IX. Given four lengths, the sides of a trapezoid, how many angles is it necessary to know in order to construct the figure? If the trapezoid is a parallelogram, how many sides and angles must we know?

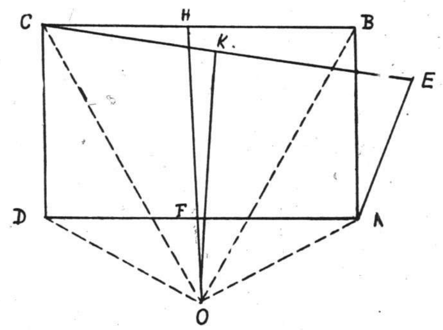
X. When a straight line cuts two parallel lines, how many angles does it make? Divide these angles into groups, each group containing those equal to one another. Name the angles.

XI. How many points are necessary to determine a circle? How many to determine a straight line through the centre of the circle?

XII. Do we mean by the following terms, *areas* or *lines*: triangle, sector, segment, circle, angle, plane?

XIII. How many degrees in the supplement of 107°. In the complement of 107°. Can you construct an angle of 410°.

Theorem: To prove that a right angle is equal to an angle greater than a right angle.



Let ABCD be a rectangle. From A draw a line AE outside the rectangle equal to AB, and making an acute angle with AB. Bisect CB in H, and through H draw HO at right angles to CB. Bisect CE in K, and through K draw KO at right angles to CE. Since CB and CE are not parallel, the line HO, KO will meet. Now triangles OCK and OEK are equal in all respects. Triangles OCH, OBH