

$$\frac{a^4(b^2 - c^2) + b^4(c^2 - a^2) + c^4(a^2 - b^2)}{a^2(b-c) + b^2(c-a) + c^2(a-b)}$$

$$= \frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$$

4. Find the values of  $x$  and  $y$  that will render the fraction

$$\frac{2z^2 + (x-a)z + 2b(x-2c)}{3z^2 + (y-b)z + 3a(y-3c)}$$

the same for all values of  $z$ .

5. If the equations  $ax^3 + bx + c = 0$ ,  $a + bx + cx^3 = 0$  be not identical, and have two roots in common, these roots are imaginary.

6. Show how to find the sum of  $n$  terms of a series in Geometric progression.

(1) Show that the sum of  $n$  terms of the series  $1 + r + (1+2r)(1+r) + (1+3r)(1+r)^2 + \dots$  is  $n(1+r)^n$ .

(2) Sum to infinity the series  $\frac{1}{2.4.6} + \frac{1}{4.6.8} + \dots$

4.6.8 6.8.10

7. Explain the notation of functions :

prove that if  $f(m) = 1 + mx + \frac{m(m-1)}{1.2}x^2 + \dots$ , then  $f(m) \times f(n) = f(m+n)$ .

Show that in the expansion of  $(1+x)^n$  the sum of the squares of the co-efficients

$$= \frac{1.2.3 \dots 2n}{(1.2.3 \dots n)^2}$$

8. Solve the equations—

(1)  $\frac{x-a}{b+c} + \frac{x-b}{a+c} + \frac{x-c}{a+b} = 3$ .

(2)  $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ .

(3)  $\frac{21x^2 - 13x + 2}{12x^2 - 7x + 1} + \frac{1}{28x^2 - 15x + 2} =$

9. Give a brief account of mathematical induction, and show that the square of a multinomial is equal to the square of each term together with twice the product of each term into the sum of all that follow it.

Find the sum of the products of the

first  $n$  natural numbers taken two and two together.

10. If  $\frac{x}{a} = y + z, \frac{y}{b} = z + x, \frac{z}{c} = x + y$ , prove

(1)  $\frac{1}{a} \cdot \frac{1}{b} \cdot \frac{1}{c} = \frac{1+a}{1-ab} \cdot \frac{1+b}{1-bc} \cdot \frac{1+c}{1-ca}$

(2)  $\frac{x^2}{a(1-bc)} = \frac{y^2}{b(1-ca)} = \frac{z^2}{c(1-ab)}$

(3)  $\frac{\sqrt{1-bc}}{a} + \frac{\sqrt{1-ca}}{b} + \frac{\sqrt{1-ab}}{c} = \frac{\sqrt{1-bc}}{a} \cdot \frac{\sqrt{1-ca}}{b} \cdot \frac{\sqrt{1-ab}}{c}$

11. AB is divided in C, so that AB, BC = AC<sup>2</sup> : from CA is cut off a part CD equal to CB ; from DC is cut off a part DE equal to DA ; from ED is cut off a part equal to EC, and so on *ad inf.* Show that the points of section continually approach a point C' such that AC' = BC.

12. Eliminate  $x, y, z$  and  $u$  from the equations

$$\begin{aligned} a_1x + b_1y + c_1z + d_1u &= 0. \\ a_2x + b_2y + c_2z + d_2u &= 0. \\ a_3x + b_3y + c_3z + d_3u &= 0. \\ a_4x + b_4y + c_4z + d_4u &= 0. \end{aligned}$$

13. A railway train travels from Toronto to Collingwood. At Newmarket it stops 7 minutes for water, and two minutes after leaving the latter place it meets a special express that left Collingwood when the former was 28 miles on the other side of New Market ; the express travels at double the rate of the other, and runs the distance from Collingwood to Newmarket in  $1\frac{1}{2}$  hour ; and if on reaching Toronto it returned at once to Collingwood, it would arrive there three minutes after the first train : find the distance between Toronto, Newmarket and Collingwood.

ANSWERS TO QUESTIONS.

NATURAL PHILOSOPHY, SECOND CLASS.

2.  $5\sqrt{3}$  lbs. The line of action of the resultant will be perpendicular to that of the 1 lb. force, and will therefore be equally in-