123. Let n and $n+2$ be the two nos. ; their prod. $n + 2n$ is less by 1 than n^2+2n+1, which is the sq. of $n+1$.

124. See Arith. p. 131, no. 323.

125. $1\frac{3}{4} - \sqrt{3} = \frac{7-4\sqrt{3}}{4} = \frac{1}{4(7+4\sqrt{3})}$ which is less than $\frac{1}{52}$ since $\sqrt{3}$ is greater than $1\frac{1}{2}$ (since the sq. of $1\frac{1}{2} = 2\frac{1}{4}$ only).

126. The sq. of 12345 exceeds that of 12344 by $2 \times (12344) + 1$; subtracting this leaves 152374336.

127. If the even no. is in the right hand the mult. gives even by even wh. is even, and odd by odd which is odd ; and the sum of odd and even is odd. If the odd no. is in the right hand the mult. gives even by odd which is even, and odd by even which is even, and the sum of even and even is even.

128. The first of the two nos. must be of one of the forms $3m$, $3m+1$, $3m+2$; and the second, of one of the forms $3n$, $3n+1$, $3n+2$, now one of the first can be taken with one of the second in 9 different ways, viz.: (denoting the nos. respectively by a, b, c, x, y, z) ax, ay, az, bx, by, bz, cx, cy, cz. In 5 of these cases ax, ay, az, bx, cx one of the nos. is divisible by 3 ; in 2 of the cases bz, cy, the sum, and in the remaining two, by, cz, the difference, is divisible by 3.

129. See Arith. p. 131, no. 335. Let $2n+1$ represent any odd no.; its sq. is $4n^2+4n+1$; the two nos. nearest the half of this sq. are $2n^2+2n$, $2n^2+2n+1$; and these are the two sides and the hyp. of a rt. angled tri. The ratio of the greater of these sides to the less is $\frac{2n^2+2n}{2n+1} = n + \frac{n}{2n+1}$, which by giving n the successive values 1, 2, 3, &c., produces the series $1\frac{1}{3}$, $2\frac{2}{5}$, &c.

130. This is equivalent to multiplying the no. by $\frac{1}{400}\left(1 + \frac{1}{11} + \frac{1}{11}\cdot\frac{1}{20} + \frac{1}{11}\cdot\frac{1}{20}\cdot\frac{1}{11} + \dots\right) = \frac{1}{400}\left(1+\frac{1}{11}\right)\left(1 + \frac{1}{220} + \frac{1}{220^2} + \frac{1}{220^3} + \dots\right) = \frac{1}{400} \times \frac{12}{11} \times \frac{1}{1-\frac{1}{220}} = \frac{1}{365}$.

314

131. This is equivalent to multiplying the no. by $\left(1+\frac{1}{3}+\frac{1}{30}+\frac{1}{300}\right) \times \frac{1}{100000} = \frac{411}{30000000}$; but $\frac{1}{73000} = \frac{411}{30000000} \times \frac{1}{1+\frac{1}{10000}} = \frac{411}{30000000} \times \left(1 - \frac{1}{10000} + \frac{1}{10000^2} - \&c.\right)$ $\therefore$ the re-