CHAPTER I.

INTRODUCTORY.

The heart-shaped curve commonly known as the *Cardioide* is an exceedingly interesting one from many points of view.

Its form, length, area, Cartesian equation and generation as an epicycloid, seem to have been first indicated by JACOB OZANAM in 1691.* Early in 1692 JAMES BERNOULLI showed the curve to be a catacaustic of a circle and further, that the catacaustic of the cardioide for a luminous cusp is the (so called) "two-cusped" epicycloid or nephroid; in the *Lectiones* (1691—92) of his brother, JOHN BERNOULLI, the cardioide was again treated as a catacaustic.

A presentation of previous results was given by DE L'HOPITAL (a pupil of JOHN BERNOULLI) in his *Analyse d. Inf. Petits etc.* 1696 p. 113-117. Similarly, we may speak of an article in 1703 by LOUIS CARRÉ, who wrote the first complete work on the integral calculus.** In 1705 CARRÉ discussed a portion of a curve which he derived in the manner of a circular conchoide, but it was DE REAUMUR, 1708, who first pointed out that CARRÉ had thus generated a part of the cardioide. The cardioide was, however, first, both named and generated in its completeness as the conchoide of a circle, by DE LA HIRE in 1707 (Mém. acad. franc.); here too, it is pointed out for the first time that the cardioide is a particular case of a Pascal Limaçon. The next additions to the cardioide properties were made by COLIN MACLAURIN. In the *Philos. Trans.* 1718 and his *Geometria Organica* 1720, he showed that the cardioide is a pedal of a circle; that the parabola is the fourth, and the cubic of Tschirnhausen (§ 32) the fifth, negative pedal of a cardioide with respect to its cusp. He showed further, that particular cases of curves defined by the equation

* To this and other dates more exact references are given in the body of the paper.

** *Méthode pour la mesure des surfaces, la dimension des solides, leurs centres de pesanteur, de percussion, et d'oscillation par l'application du calcul intégral. Paris 1700.*