N.B.—These must carefully be distinguished from $(\sin x)^2$, $(\sin x)^{-1}$, $(\sin x)^0$, which come under the former or following head, and are frequently, though inaccurately, written as above.

(3). Indices denoting any function whatever,—

Example (1): Let $f^1(x)$ be the differential of $x = dx$, $f^0(x)$ is x, $f^{-1}(x)$ is $d^{-1}(x)$ meaning that which, if differentiated, will give x—in other words the integral of x. $f^{-2}(x)$ is $d^{-2}x$ that which, if differentiated *twice*, will give x, or the *second* integral of x, and so on.

It will be observed that this illustration shews clearly that a definite meaning is attached to the inverse symbol, for although our analysis may not be sufficient to enable us, in any special case, to integrate the required number of times, yet the operation is not only *conceivable* but never beyond the bounds of *possibility*, and may be *practicable*, and, what is more, may in every case be performed independently of our knowledge of the results of differentiation.

Example (2): Let $f(x) = x + \frac{1}{x}$

$$f^0 x = x$$

$$f^{-1}(x) = \frac{x}{2} \pm \frac{\sqrt{x^2 - 4}}{2}$$

For performing the function f on this we get,—

$$\frac{x \pm \sqrt{x^2 - 4}}{2} + \frac{2}{x \pm \sqrt{x^2 - 4}} = \frac{2x^2 - 4 + 2x\sqrt{x^2 - 4}}{2(x \pm \sqrt{x^2 - 4})}$$
$$+ \frac{4}{2(x \pm \sqrt{x^2 - 4})}$$
$$= \frac{2x}{2} = x$$

And similarly,

$$f^{-2}(x) = \frac{x \pm \sqrt{x^2 - 4} \pm \sqrt{2(x^2 - 10 \pm \sqrt{x^2 - 4})}}{4}$$

which may be verified. Beyond this point, the analysis fails to give the inverse function, though equations may be found to determine them. To take one more example,—