also,

$$\left\{\frac{1+x+2yz}{1-x}\right\}^{\frac{1}{2}}+\left\{\frac{1+y+2zx}{1-y}\right\}^{\frac{1}{2}}$$
$$+\left\{\frac{1+z+2xy}{1-z}\right\}^{\frac{1}{2}}=\frac{x+y}{1-z}+\frac{y+z}{1-x}+\frac{z+x}{1-y}.$$

Given identity becomes

$$z(1-x^2-y^2+x^2y^2)^{\frac{1}{2}}+\ldots\ldots=1+xyz,$$

or $z(x^2y^2+2xyz+z^2)^{\frac{1}{2}}+\ldots\ldots=1+xyz,$

i.e., $x^2+y^2+z^2+2xyz=1,$

we have $\left\{\frac{1+x+2yz}{1-x}\right\}^{\frac{1}{2}}=\frac{y+z}{1-x},$

for $(1+x+2yz)(1-x)=(y+z)^2$

gives $x^2+y^2+z^2+2xyz=1.$

2. Solve the equations

(1) $x^2+4xy+y^2=13=8xy-7x^2+y^2.$

(2) $(1+x)^{\frac{2}{n}}-(1-x)^{\frac{2}{n}}=(1-x^2)^{\frac{1}{n}}.$

(1) Subtracting $x=0$, $\therefore y=\pm\sqrt{13},$ and $2x=y$, whence $x=\pm 1$, $y=\pm 2$.

(2) Dividing both sides of the equation by right hand member, it becomes, putting $\left\{\frac{1+x}{1-x}\right\}^{\frac{1}{n}}=y,$

$y-y^{-1}=1,\ \ y=\frac{\pm\sqrt{5}+1}{2}$, whence x.

3. If a be a root of the equation $f(x)=0$, then $x-a$ is a factor of $f(x)$.

The equation $4x^3-52x^2+49x-12=0$ has two equal roots; find all the roots.

The roots of the equation

$$x^4-10x^3+32x^2-38x+15=0$$

are of the form $\alpha+1$, $\alpha-1$, $\beta+2$, $\beta-2$; find all the roots.

Bookwork.

(1) $4x^3-52x^2+49x-12=0$
$=(x-\alpha)^2(x-\beta).$

Equate coefficients

$\alpha=\frac{1}{2}$ or $\frac{49}{6}$, $\beta=12$ or $\frac{13}{3}$,

(2) Similarly, $\alpha=2$ or $\frac{13}{3}$, $\beta=3$ or $\frac{13}{3}$,

4. Sum the series

$$1^2+2^2+3^2+\ldots\ldots+n^2.$$

Bookwork.

5. Shew how to find the sum of an Arithmetical Progression, having given the first term, common difference, and number of terms.

Sum to n terms the series whose first term is a, and the successive differences b, $2b$, $3b\ldots\ldots(n-1)b$.

Bookwork; $S=na+\frac{n(n-1)b}{2}.$

6 Sum to n terms the series....

$$1+3x+5x^2+7x^3+\ldots$$

If the natural numbers be divided into groups 1, 2+3, 4+5+6, etc., find the sum of the n^{th} group, also the sum of the first n groups, and thence deduce the sum of $1^3+2^3+3^3+\ldots\ldots+n^3$.

Let $S=1+3x+5x^2+\ldots\ldots+(2n-1)x^n.$

$(S-1)n=x+3x^2+\ldots\ldots+(2n-3)x^n$
$+(2n-1)x^{n+1}.$

Subtract and sum resulting G. P. when

$$S=1+\frac{2x(1-x^n)}{(1-x)^2}-\frac{(2n-1)x^n}{1-x}.$$

1st term of n^{th} group

$=1+1+2+3+\ldots\ldots+n-1$

$=1+\frac{n(n-1)}{2}.$

$\therefore$ Sum of n^{th} group

$=\left(1+\frac{n(n-1)}{2}\right)+\left(1+\frac{n(n-1)}{2}\right)+1$

$+\left(1+\frac{n(n-1)}{2}\right)+2+\ldots$ to n terms

$=\frac{n^3+n}{2}.$

$\therefore S$ (the sum of 1st n groups)

$$=\frac{1}{2}\left\{\frac{n(n+1)}{2}+\left(\frac{n(n+1)}{2}\right)^2\right\}$$

by method of indeterminate coefficients, whence $\Sigma n^3=\left(\frac{n(n+1)}{2}\right)^2.$

7. Find the number of combinations of n things, r together.

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