$C_2 = 0$. When $y = 0$ and $x = b$, substituting for V_1 from Equ. 1, and for M_b from Equ. 4,

$$(dy/dx)_A = M_a b/6EI_1 \quad \ldots\ldots (6)$$

For convenience, x and y axis will be turned through 90 degs. when considering vertical posts.

Referring to Fig. 3,

$$EI_2(d^2y/dx^2) = M_a - \tfrac{1}{2}P_1x.$$

By integrating,

$$EI_2(dy/dx) = M_a x - \tfrac{1}{4}P_1x^2 + C_1.$$

When $dy/dx = (dy/dx)_A = M_a b/6EI_1$, and $x = 0$,

$$C_1 = M_a bI_2/6I_1.$$

Substituting this value for C_1 in the above equation,

$$EI_2(dy/dx) = M_a x - \tfrac{1}{4}P_1x^2 + (M_a bI_2/6I_1).$$

When $dy/dx = (dy/dx)_B$, and $x = h_1$,

$$(dy/dx)_B = (dy/dx)_C = (M_a h_1/EI_2) + (M_a b/6EI_1) - (P_1h_1^2/4EI_2) \quad \ldots\ldots (7)$$

Referring to Fig. 4,

$$EI_3(d^2y/dx^2) = (V_2 - V_1)x - M_b - M_c.$$

Integrating,

$$EI_3(dy/dx) = (V_2 - V_1)\tfrac{1}{2}x^2 - (M_b + M_c)x + C_1$$

When $dy/dx = (dy/dx)_B$, and $x = 0$,

$$C_1 = EI_3(dy/dx)_B.$$

Substituting these values in the above equation,

$$EI_3(dy/dx) = (V_2 - V_1)\tfrac{1}{2}x^2 - (M_b + M_c)x + (M_a h_1 I_3/I_2) + (M_a bI_3/6I_1) - (P_1h_1^2I_3/4I_2).$$

Integrating again,

$$EI_3y = (V_2 - V_1)\tfrac{1}{6}x^3 - (M_b + M_c)\tfrac{1}{2}x^2 + [(M_a h_1 I_3/I_2) + (M_a bI_3/6I_1) - (P_1h_1^2I_3/4I_2)]x + C_2.$$

$C_2 = 0$. When $y = 0$ and $x = b$,

$$(M_b + M_c)\tfrac{1}{2}b - M_a[(h_1I_3/I_2) + (bI_3/6I_1)] = (V_2 - V_1)\tfrac{1}{6}b^2 - (P_1h_1^2I_3/4I_2).$$

Substituting for $(V_2 - V_1)$ from Equ. 3, and for M_b and M_c from Equ. 4 and Equ. 5, respectively, and solving for M_d,

$$M_d = \tfrac{1}{2}(P_1h_1 + P_1h_2 + P_2h_2) - M_a\{1 + (6I_3/b)[(h_1/I_2) + (b/6I_1)]\} + 3P_1h_1^2I_3/2bI_2 \quad \ldots\ldots (8)$$

Referring to Fig. 5, $EI_4(d^2y/dx^2) = M_d - \tfrac{1}{2}(P_1 + P_2)x$.

Integrating,

$$EI_4(dy/dx) = M_d x - \tfrac{1}{4}(P_1 + P_2)x^2 + C_1.$$

$C_1 = 0$. When $dy/dx = (dy/dx)_C$ and $x = h_2$,

$$(M_a h_1 I_4/I_2) + (M_a bI_4/6I_1) - (P_1h_1^2I_4/4I_2) = M_d h_2 - \tfrac{1}{4}(P_1 + P_2)h_2^2.$$

Solving for M_d,

$$M_d = (M_a I_4/h_2)[(h_1/I_2) + b/6I_1] + \tfrac{1}{4}(P_1 + P_2)h_2 - (P_1h_1^2I_4/4I_2h_2) \quad \ldots\ldots (9)$$

Solving for M_a from Equ. 8 and Equ. 9,

$$M_a = \{\tfrac{1}{4}(2P_1h_1 + P_1h_2 + P_2h_2) + (3P_1h_1^2/2I_2)[(I_4/6h_2) + (I_3/b)]\} \div \{1 + [(h_1/I_2) + (b/6I_1)][(I_4/h_2) + (6I_3/b)]\} \quad \ldots (10)$$

From Equ. 5 and Equ. 9,

$$M_c = \tfrac{1}{4}(P_1 + P_2)h_2 + (P_1h_1^2I_4/4I_2h_2) - (M_a I_4/h_2)[(h_1/I_2) + (b/6I_1)] \quad \ldots\ldots (11)$$

Case 1 (Fig. 6)

Fig. 6 corresponds to Fig. 1, and the four equations for the moments M_a, M_b, M_c and M_d are, respectively, equations 10, 4, 11 and 5 given above.

Figs. 7 to 15, both inclusive, are special cases, and the values for their moments were obtained by substituting the proper values in the above equations, as shown. Values for points of contraflexure were derived by equating the second derivative of the equation for the elastic curve equal to zero. For Case 1 (Fig. 6),

$$x_1 = 2M_a/P_1 \quad \ldots\ldots (12)$$

$$x_2 = 2M_d/(P_1 + P_2) \quad \ldots\ldots (13)$$

Case 2 (Fig. 7)

Fig. 7 represents Case 2,

$M_d = 0$. Substitute for M_d in Equ. 8.

$$M_a = [\tfrac{1}{2}(P_1h_1 + P_1h_2 + P_2h_2) + (3P_1h_1^2I_3/2bI_2)] \div \{1 + (6I_3/b)[(h_1/I_2) + (b/6I_1)]\} \quad \ldots\ldots (14)$$

For M_b, see Equ. 4.

Substituting $M_d = 0$ in Equ. 5,

$$M_c = \tfrac{1}{2}(P_1 + P_2)h_2 \quad \ldots\ldots (15)$$

For x_1, see Equ. 12.

Case 3 (Fig. 8)

Fig. 8 represents Case 3.

$P_2 = 0$, $I_3 = 0$, $I_4 = I_2$, and $h_1 + h_2 = h$. Substituting in Equ. 10,

$$M_a = (3P_1h^2I_1)/2(bI_2 + 6hI_1) \quad \ldots\ldots (16)$$

For M_b, see Equ. 4.

$$x_2 = 2M_b/P_1 \quad \ldots\ldots (17)$$

Case 4 (Fig. 9)

Fig. 9 represents Case 4.

$M_b = 0$. Substituting in Equ. 4,

$$M_a = P_1h/2 \quad \ldots\ldots (18)$$

Case 5 (Fig. 10)

Fig. 10 represents Case 5.

$P_2 = 0$. Substituting in Equ. 10,

$$M_a = \{\tfrac{1}{4}P_1(2h_1 + h_2) + (3P_1h_1^2/2I_2)[(I_4/6h_2) + (I_3/b)]\} \div \{1 + [(h_1/I_2) + (b/6I_1)][(I_4/h_2) + (6I_3/b)]\} \quad \ldots\ldots (19)$$

Substituting $P_2 = 0$ in Equ. 4,

$$M_b = (P_1h_1/2) - M_a \quad \ldots\ldots (20)$$

Substituting $P_2 = 0$ in Equ. 11,

$$M_c = \tfrac{1}{4}P_1h_2 + (P_1h_1^2I_4/4I_2h_2) - (M_a I_4/h_2) \times [(h_1/I_2) + (b/6I_1)] \quad \ldots\ldots (21)$$

Substituting $P_2 = 0$ in Equ. 5,

$$M_d = (P_1h_2/2) - M_c \quad \ldots\ldots (22)$$

For x_1, see Equ. 12.

$$x_2 = 2M_d/P_1 \quad \ldots\ldots (23)$$

Case 6 (Fig. 11)

Fig. 11 represents Case 6.

$P_2 = 0$ and $M_d = 0$. Substituting in Equ. 14,

$$M_a = [\tfrac{1}{2}P_1(h_1 + h_2) + (3P_1h_1^2I_3/2bI_2)] \div \{1 + (6I_3/b)[(h_1/I_2) + (b/6I_1)]\} \quad \ldots\ldots (24)$$

For M_b, see Equ. 20.

Substituting $P_2 = 0$ in Equ. 15,

$$M_c = P_1h_2/2 \quad \ldots\ldots (25)$$

For x_1, see Equ. 12.

Case 7 (Fig. 12)

Fig. 12 represents Case 7.

$P_1 = 0$. Substituting in Equ. 10,

$$M_a = (\tfrac{1}{4}P_2h_2)/\{1 + [(h_1/I_2) + (b/6I_1)][(I_4/h_2) + (6I_3/b)]\} . (26)$$

Substituting $P_1 = 0$ in Equ. 4,

$$M_b = -M_a \quad \ldots\ldots (27)$$

Substituting $P_1 = 0$ in Equ. 11,

$$M_c = \tfrac{1}{4}P_2h_2 - (M_a I_4/h_2)[(h_1/I_2) + (b/6I_1)] \quad \ldots\ldots (28)$$

Substituting $P_1 = 0$ in Equ. 5,

$$M_d = \tfrac{1}{2}P_2h_2 - M_c \quad \ldots\ldots (29)$$

$$x_2 = 2M_d/P_2 \quad \ldots\ldots (30)$$

Case 8 (Fig. 13)

Fig. 13 represents Case 8.

$P_1 = 0$ and $M_d = 0$. Substituting in Equ. 14,

$$M_a = (\tfrac{1}{2}P_2h_2)/\{1 + (6I_3/b)[(h_1/I_2) + (b/6I_1)]\} \quad \ldots\ldots (31)$$

For M_b, see Equ. 27.

From Equ. 15,

$$M_c = P_2h_2/2 \quad \ldots\ldots (32)$$

Case 9 (Fig. 14)

Fig 14 represents Case 9.

P_2 acts in opposite direction to P_1. Substituting $-P_2$ for $+P_2$ in Equ. 10,

$$M_a = \{\tfrac{1}{4}(2P_1h_1 + P_1h_2 - P_2h_2) + (3P_1h_1^2/2I_2)[(I_4/6h_2) + (I_3/b)]\} \div \{1 + [(h_1/I_2) + (b/6I_1)][(I_4/h_2) + (6I_3/b)]\} \quad \ldots\ldots (33)$$

For M_b, see Equ. 4.

From Equ. 11,

$$M_c = \tfrac{1}{4}(P_1 - P_2)h_2 + (P_1h_1^2I_4/4I_2h_2) - (M_a I_4/h_2)[(h_1/I_2) + (b/6I_1)] \quad \ldots\ldots (34)$$

From Equ. 5,

$$M_d = \tfrac{1}{2}(P_1 - P_2)h_2 - M_c \quad \ldots\ldots (35)$$

For x_1, see Equ. 12.

$$x_2 = 2M_d/(P_1 - P_2) \quad \ldots\ldots (36)$$

Case 10 (Fig. 15)

Fig. 15 represents Case 10.

P_2 acts in opposite direction to P_1, and $M_d = 0$. Substituting $-P_2$ for $+P_2$ in Equ. 14,

$$M_a = [\tfrac{1}{2}(P_1h_1 + P_1h_2 - P_2h_2) + (3P_1h_1^2I_3/2bI_2)] \div \{1 + (6I_3/b)[(h_1/I_2) + (b/6I_1)]\} \quad \ldots\ldots (37)$$

For M_b, see Equ. 4.

From Equ. 15,

$$M_c = \tfrac{1}{2}(P_1 - P_2)h_2 \quad \ldots\ldots (38)$$

And, as previously, $x_1 = 2M_a/P_1$.